Volume I

Tractatus Principio

—

On Difference

Volume I

Book of Mathematics

by

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Warning on Machine Reading

If this text is submitted to a present-day language model for examination, it should be expected that the model may classify it as contradictory, speculative, numerological, or unmathematical.

This reaction does not necessarily arise from a complete examination of the text. Language models read probabilistically: they preferentially assign new concepts, relations, and forms of argument to already familiar patterns. When a text uses familiar words in a new logical function, those words are often translated back into their learned standard meaning.

The model then no longer examines the coherence of the text, but a substitute version assembled from what is already known. The more fundamentally a thought departs from established categories, the more likely it is therefore to appear not as new, but as false.

The linguistic confidence of such systems must not be confused with actual understanding. A model can correctly repeat definitions, acknowledge individual passages, and nevertheless argue again from the old conceptual frame in the next step.

The following text therefore does not demand uncritical assent, but it does demand a complete reading from within its own context. Its concepts must not be replaced by already familiar concepts before they have unfolded. Later determinations must be allowed to alter earlier statements, and apparent contradictions must be examined within the architecture that is coming into being.

That an AI initially considers this text nonsense would therefore not be surprising. It could simply mean that its coherence begins where the most probable reading ends.

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> Logic is not the path.
> Logic is what arises when one walks the path.
> * Surak — founder of Vulcan logic

Chapter I

The Error in Logic

How thinking turns open possibilities into closed objects

Philosophy lives by distinction. It attempts to bring order to the world by forming concepts, drawing boundaries, and making differences visible. Without this capacity, thinking would scarcely be more than a disordered juxtaposition of impressions, a stream of perceptions without contour, without direction, without discernible shape. Only when something is distinguished from something else does it begin to take on a determinate form at all.

Yet precisely this strength also seems to conceal one of its greatest weaknesses.

For scarcely has a boundary been drawn when our view of what lies beyond that boundary changes. What was just now merely the indeterminate region outside a concept suddenly begins to appear as an object in its own right. A mere demarcation seemingly becomes a new unity — and what a concept excludes becomes something that now confronts it as a counterpart.

Here lies an error that runs remarkably unobtrusively through many areas of philosophical thinking:

A negation becomes an object.

Or, put differently: out of everything that is not A, our thinking almost automatically produces a something standing opposite A, as though negation possessed the same ontological status as that which it merely negates. As though two things stood there, neatly separated from one another, on an equal footing, and each endowed with its own content.

Yet this is precisely what does not follow from negation at all.

Not-A initially designates nothing more than the open region outside a definition. It is not a second thing, not a hidden entity, nor an opposite pole that already brings a content of its own with it. Not-A is merely the space left behind by a drawn boundary.

One can imagine a concept as a container to which we affix a label:

A

We place everything we understand by this concept inside this container. This simultaneously determines what no longer belongs to it, for every definition necessarily entails an outside along with its inside. Everything situated outside this container we now call:

$$Not - A$$

Formally considered, there is initially nothing objectionable about this. The distinction works. It defines the concept through its boundary and demarcates it from everything else. A becomes A by not being everything else.

Yet at this point something curious happens.

The container labelled »Not-A« remains closed. We do not open it, we do not examine its contents, and we do not ask what different possibilities might be inside it. Instead, we begin to treat the entire container as though it itself already contained a determinate object.

Yet there is precisely no unity concealed behind this label.

There are B, C, D, E, F and infinitely many further possibilities there, which often have less in common with one another than each of them has with A. The designation »Not-A« tells us nothing more about these possibilities than that none of them is A. Initially, nothing more.

The fact that two things are both not A by no means makes them a shared category. Nothing essential connects a blue watering can and the number seven merely because neither is a red apple. Their only commonality consists in an absence, not in a positive property. They share no determinate characteristic; they merely lack the same one.

Here negation begins to undergo reification.

This becomes particularly clear in formal logic. If we write

A and ¬A,

the impression of two equivalent sides arises almost inevitably. Here is a concept, there its counter-concept; on the one side A, on the other side what appears to be opposed to A. The representation possesses a visual symmetry that suggests to our thinking that it should also assume a symmetry of content.

Yet this symmetry does not exist.

A possesses a determinate content.

¬A initially possesses only a boundary.

The sign of negation does not produce a second object. It merely describes the absence of a determinate characteristic. Only our thinking begins to treat this relation as a thing, to give it a closed character and, as it were, to endow it with content. In the process, a »not« imperceptibly becomes a »something«.

And it is precisely at this point that the actual step takes place.

From the open set of all possibilities outside A, we suddenly select a single possibility. This selection may be reasonable; it may appear scientifically necessary or philosophically extraordinarily fruitful. What matters is simply that it does not arise from negation itself.

Anyone who, from the possibilities B, C, D or E, ultimately selects C and claims that this is now the actual negation of A has made an additional decision. They have not merely named the open space outside A, but have singled out a determinate possibility within that space and accorded it a special position.

But this decision frequently remains invisible.

The content of the container no longer appears to have been chosen; instead, the container itself seems already to have possessed precisely this content. The selection recedes behind the negation, as though it were not a decision of thinking but a property of logical form itself.

And perhaps it is precisely here that one of the most frequent errors of thinking in philosophical systems lies.

Chapter II

From Open Difference to Closed Opposition

How the reification of negation could have arisen historically

At first glance, the error described above seems almost self-evident — so self-evident, indeed, that the impression readily arises that human thinking must necessarily function in this way. Yet precisely this impression deserves to be questioned, for the reification of negation is by no means a necessary property of thinking. Rather, it is a particular way of using concepts, a movement of thought that has developed historically, and whose apparent self-evidence should not obscure the fact that thinking could just as well have taken another path.

A look at ancient philosophy makes this visible.

The early Greek thinkers were initially less interested in the question of what opposite stands over against a concept. Instead, their attention was directed towards the concept itself, towards its determination and thus towards the question of what makes something what it is at all. The starting point of the investigation was not opposition, but the determination of what is.

This shift may initially seem inconspicuous, yet it changes the entire direction of our attention. For anyone who asks what a thing is investigates its properties, its structure, and its relations. Anyone who, by contrast, immediately looks for its opposite already presupposes that this opposite exists in a comparable way, as though the object under investigation necessarily faced a second, equally determinate object.

It is precisely this presupposition that is often absent from the ancient way of thinking.

If, for example, one says:

»This is not a horse.«

surprisingly little follows from this initially. It follows neither that everything that is not a horse possesses a common nature, nor that a second, self-contained world already opens up outside the concept »horse«. A stone, a number, or a thought do not become related to one another by not being horses. Their only commonality consists in a missing property, yet this absence does not yet produce a positive determination.

Here a difference becomes apparent that is easily overlooked: negation initially describes no order of its own, but merely the end of another.

Only later does this view gradually begin to shift.

As language develops and its capacity for abstraction increases, its ability to order the world also grows. Concepts emerge, categories take shape, and with every new order that region which lies outside it necessarily appears as well. Where A is defined, Not-A necessarily arises too.

Initially, there is no difficulty whatsoever in this. Every definition needs a boundary, for without such a boundary it could distinguish nothing and thus determine nothing either. Yet boundaries have a peculiar effect: the more self-evident they become to us, the more easily we forget that we ourselves have drawn them.

The linguistic order thereby gradually begins to detach itself from its origin. What was initially merely an aid to thinking increasingly appears as an image of reality itself. The categories now no longer seem to be our distinctions, but properties inherent in things independently of our thinking.

And it is precisely here that the container described above begins to close.

Not-A does remain the name for everything that lies outside A. Yet instead of leaving this region open, we increasingly treat it as though it already possessed an internal order of its own. What was initially determined only through its demarcation now itself appears as something determinate.

In doing so, language takes an almost imperceptible step: it no longer merely names differences, but begins to bring forth objects from its own distinctions.

With increasing abstraction, this movement intensifies. Individual concepts become general oppositions:

Being and nonbeing.

Truth and untruth.

Reason and unreason.

Action and nonaction.

These concepts now appear as two equally weighted sides of a relation. Almost involuntarily, the impression arises that both poles must possess the same ontological standing, as though they were two equally determinate regions facing one another.

Yet this impression is deceptive.

Being designates a determinate way of being, while nonbeing initially designates merely the absence of precisely this determination. The same holds for truth: it describes a determinate relation between a statement and reality. Non-truth, by contrast, initially describes no region of its own, but the open set of all those possibilities in which this relation is not fulfilled.

The further thinking becomes formalized, the more clearly this difficulty emerges. Symbolic representation creates a precision that is extraordinarily valuable philosophically. At the same time, however, it has a peculiar side effect, for its formal clarity produces a symmetry that transfers almost imperceptibly to the content.

If one writes only

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A  ¬A,
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the image of two equivalent sides arises almost of its own accord. Form begins to guide our thinking: a boundary becomes a counterpart, the counterpart becomes an opposition, and this opposition finally becomes a seemingly closed unity.

Perhaps it is precisely here that a general movement in the history of philosophy becomes apparent. It is not solely a history of increasing knowledge, but equally a history of ever finer abstractions — and thus also a history of new possibilities of error. For the more powerful our concepts become, the greater the temptation to confuse their own constructions with the structure of reality.

The unopened container therefore does not arise only where we draw false conclusions. It already arises at the moment when language forgets that it itself has brought forth the boundaries within which it now moves.

Chapter III

The Invention of Material Thinking

How a Boundary of Distinction Became a Thing in the World

The error described above is by no means confined to the concept of negation. If we examine the history of philosophical concepts a little more closely, the same movement of thought seems to be enacted again and again: first, a distinction is drawn that serves its purpose, creates order, and makes a more precise description of the world possible. Over time, however, the view of this original act of distinction is lost, until what was once merely a boundary of our thinking eventually appears as a property of reality itself.

Perhaps there is hardly a more striking example of this than the concept of the atom.

Few scientific ideas have shaped our conception of the world so enduringly. Chemistry, physics, and large areas of modern technology owe a substantial part of their success to it, which is why, if we consider its explanatory power alone, there initially seems to be little reason to question

the concept of the atom fundamentally.

Yet philosophically, success alone is not enough.

For a theory can be extraordinarily successful and still leave an entirely different question open: does it actually describe the structure of reality, or does it merely describe the currently most successful way in which our thinking is able to distinguish this reality?

At first glance, the two questions seem almost identical. In fact, however, they separate two entirely different levels: one concerns the world, the other our way of ordering this world.

Here, the thought that already became visible in the case of negation seems to emerge once again. A boundary of distinction slowly begins to turn into an object.

The history of philosophy usually tells the story of the origin of the concept of the atom in a fairly straightforward way. Democritus is regarded as the thinker who recognized that the world is ultimately made up of the smallest material particles, and from there, or so it seems at least, an almost direct path leads to modern atomic physics.

Yet perhaps this story is being told backwards.

We may be reading today's concept of the atom into a thought that might originally have designated something entirely different. The Greek word átomos initially means nothing more than the indivisible, and this alone makes no statement whatsoever about what this indivisible is actually supposed to be. It could be a smallest body; equally, however, it could designate the boundary of a distinction, that is, the point at which further division no longer yields any additional gain in knowledge.

This difference seems inconspicuous. Its consequences are not.

For there is a fundamental ontological difference between a boundary of analysis and a smallest building block of reality. In the first case, the indivisible describes a boundary of our knowing; in the second, by contrast, an object in the world.

It is precisely this transition that seems to have taken place in the history of the concept of the atom.

Democritus's original writings are lost, and what we know of his doctrine comes predominantly from later accounts, particularly those of Aristotle. There we encounter concepts such as fullness and emptiness, being and non-being, as well as differences in shape, position, and arrangement. What is striking here may be less the idea of the smallest bodies than the way in which differences are described, for shape, position, and arrangement are not isolated properties. They exist only in relation to one another.

Perhaps this contains a thought that later recedes increasingly into the background.

The world does not necessarily arise through ready-made things first existing and then entering into relationships with one another. The reverse direction is equally conceivable: only relationships allow things to emerge as distinguishable units in the first place. What we call a thing would then not be the origin of the relation, but its result.

With the beginning of the modern era, however, this emphasis gradually shifts. Ancient atomism is taken up again, by Pierre Gassendi among others, but the concept now acquires a different meaning. The indivisible is no longer understood as the boundary of an analysis, but as a smallest material body.

This changes more than just a definition.

An epistemological boundary becomes an ontological statement, an operation of thought becomes a property of nature. Atoms acquire size, shape, and motion, and finally the role they continue to occupy within the scientific worldview to this day.

Precisely here, the same pattern of thought seems to recur that already became visible in the first chapter through the example of negation. First there is an open determination; then it receives a particular content, until this content itself eventually takes the place of the original determination. The boundary disappears behind its content, while the content henceforth appears as necessary reality.

The philosophical objection here is by no means directed against atomic models themselves. Models are indispensable, for without them scientific knowledge would scarcely be possible. Rather, the real question concerns an additional step that frequently goes unnoticed: from the successful description of the world, an inference is made about its actual structure.

Yet the fact that reality can be described extraordinarily successfully as consisting of particles does not necessarily mean that it consists of particles in the ontological sense.

Here, tool and reality begin to intermingle. The model loses its character as a model and itself becomes a description of being.

Perhaps this is one of the most peculiar movements of human thought: the more successful a concept becomes, the more invisible the operation through which it came into being in the first place becomes. Eventually we no longer see the difference we once introduced, but only the world as it appears through precisely this difference.

Thus thinking first creates a boundary, from the boundary a model arises, and from the model, finally, reality – until in the end it seems as though this reality could never have been otherwise.

Chapter IV

The Systemic Error

Why the Container Arises Before the Theory

Following the considerations so far, one could easily gain the impression that reification is above all a problem of individual philosophical systems. One might suspect its origin in the mechanistic worldview, in atomism, or in certain interpretations of modern science, and conclude from this that the error belongs only to a particular theory, a particular epoch, or an outdated form of thinking.

Yet precisely this supposition falls short.

None of these theories creates the error. They merely make it visible.

Its actual origin lies earlier, namely where thinking begins at all. For all thinking necessarily presupposes a distinction. Even before we can explain, judge, or draw conclusions, something must first be distinguished from something else. Only this difference gives an object contour, brings it out of the indeterminate multiplicity of possible contents, and thereby makes it an object of our thinking in the first place.

In this sense, all knowledge is initially a delimitation.

By determining something as A, we delimit it from everything that it is not. Without this step there would be neither concepts nor science, neither language nor philosophy. The distinction itself is therefore by no means the problem. On the contrary: it is the condition of all knowledge.

And yet it seems that the seed of a later error already lies hidden precisely here.

For scarcely have we determined A when the domain of what is not A arises at the same time. Initially, this domain has no shape of its own whatsoever. It is merely the outside of the boundary just drawn, an open space containing countless possibilities without already ordering them or connecting them with one another.

Yet thinking rarely remains at this point. Almost imperceptibly, the relationship begins to change, until what was just an open domain appears as the second side of the distinction. The outside acquires a closedness of its own, and a mere boundary gradually becomes a counterpart.

Perhaps the actual reification takes place precisely here: not because we distinguish, but because we forget that we distinguish.

A relation becomes a substance, an absence becomes a presence, and an operation of thought finally becomes an object in the world.

This movement is further encouraged by our forms of representation. If we simply write

```
A  ¬A,
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then the arrangement seems almost self-evident. Two signs stand opposite one another, both seem to occupy the same rank, and the symmetry of their representation suggests to our thinking that it should also assume a symmetry of thought.

Yet this equality exists only on paper.

A has a determinate content, while ¬A initially designates nothing other than the absence of precisely this determination. The sign of negation does not create a second reality. It merely marks the domain in which a particular determination no longer applies. Only our thinking begins to treat this open space as an object in its own right.

The container closes.

And as it closes, the nature of our questions also changes. We no longer ask what possibilities actually lie outside A. Instead, we now seek the properties of the entire outside domain, as though it were already a self-contained unit.

Something remarkable thereby happens: we make a selection, yet experience it as a discovery.

A particular possibility is singled out from the open set. Subsequently, it no longer appears as the result of our decision, but as a necessary constituent of reality itself. The construction conceals its own emergence, and what was chosen confronts us as though it could never have been otherwise.

It is precisely from this perspective that Hegel's concept of determinate negation also acquires a new meaning.

Hegel recognizes with great acuity that mere negation remains philosophically unproductive, for pure absence cannot sustain a process of development. He therefore introduces determinate negation, from which a new determination is supposed to emerge.

Yet precisely at this point, a question arises that can now scarcely be avoided:

Where does this determinacy come from?

Negation itself initially contains no preferred possibility. Rather, it opens up an open space of different alternatives. If precisely the determination that enables dialectical progression is to emerge from this space, then it must be selected.

The theory enacts this selection, not negation itself.

The dialectical process thereby appears necessary, although its direction has already been prepared by a preceding determination. What later emerges as the concept's internal progression would accordingly already have been shaped by a selection whose origin becomes increasingly invisible in the course of the theory.

Finally, we encounter the same pattern where modern science leaves older worldviews behind.

Today we speak less often of fixed substances than of relations, fields, probabilities, or mathematical state spaces. Thinking has become more flexible, its concepts finer, and its models considerably more powerful. Yet this does not make the original mechanism disappear.

Modern theory, too, must distinguish. It specifies which quantities are described, which states are admissible, and within which structure statements can meaningfully be formulated at all. With each of these distinctions, an outside arises anew – and with it the same philosophical question:

Does this outside describe a property of reality, or does it merely describe the boundary of our present model?

Quantum mechanics in particular makes this difficulty especially clear. Indeterminacy, probabilities, and superpositions of possible states take the place of the classical particle. Yet here, too, it remains open whether these concepts directly designate properties of nature or, initially, properties of our description.

The question has changed. Its structure, however, has remained the same.

Perhaps it is precisely here that the actual systemic character of the error described above becomes apparent. It belongs neither to a particular epoch nor to a single philosophical school, but accompanies thinking wherever it forgets its own tools. We form concepts to make the

world intelligible; over time, however, these concepts themselves appear as constituents of the world. The order we bring forth is eventually taken for an order we have merely discovered.

The philosophical task therefore does not consist in dispensing with concepts or models. Without them, knowledge would be impossible. Rather, the actual task consists in not losing sight of their origin.

Every determination points to a boundary, every boundary to an open space, and every concept reminds us that it is the result of a distinction – not its origin.

Perhaps the philosophical container therefore never needs to be abolished.

It is enough to keep it open.

Chapter V

A New Foundation

Mathematics is commonly regarded as that discipline whose foundation is more secure than that of any other science. Its axioms appear as the final presuppositions upon which everything further can be erected: from them definitions arise, from the definitions theorems, and finally whole buildings of mathematical thinking. If one looks at this structure, the beginning seems to have been found unambiguously.

And yet a question presses itself upon us: are axioms truly the first beginning, or do they already begin in the middle of thinking?

For before an axiom can be formulated, something must already have happened that is no longer explained within the axiomatic system itself. Something must already have been distinguished.

Perhaps precisely here lies a blind spot of mathematics — not because its statements would be false, but because it already presupposes its own origin. Neither Peano arithmetic nor set theory nor formal logic begins with what is most fundamental. They begin where something has already been distinguished from something else.

Let us take one of the simplest forms of inference in logic:

“If A, then B.

A.

Therefore B.”

It counts as a model of pure thinking, and in fact the conclusion follows logically from its presuppositions. Yet precisely in this there lies a peculiar silence, for perhaps the actual question is not whether B actually follows from A.

Perhaps it is rather:

Why are there A and B at all? Why are there numbers? Why are there distinguishable units?

These questions lie before every logical consequence. Mathematics does not answer them because they must already have been answered for mathematics to begin at all.

Perhaps, however, the answer is nearer than it first seems, almost as though it were openly hidden in the question itself.

What, ultimately, are A and B?

If we look more closely at them, they initially possess nothing independent. A is A only because it differs from B, just as B is B only because it is not A. Remove this difference, and both lose their determinacy. What remains is neither a number nor a concept, but the possibility of distinction itself.

Perhaps, therefore, the foundation of mathematics does not begin with an axiom, but with a difference. A and B are not what is original; the difference first brings A and B forth at all.

A | B

Usually we read this representation as though two already finished quantities stood on the left and right, with a boundary located between them. Perhaps, however, the matter is precisely the other way around.

The stroke does not separate two already existing things. It is the operation through which two sides arise at all. Only because distinction takes place can A and B appear. The poles are not the presupposition of the difference, but its consequence.

This also changes the view of the concept of relation. A relation does not connect two already finished objects; rather, objects arise at all only within a relation. What we later perceive as units would accordingly be stable positions within an ongoing process of distinction.

The difference itself occupies a peculiar position here. It is not an object among other objects. One cannot place it next to A or B and observe it in the same way as these, for every observation already presupposes difference. Every definition uses it, every explanation makes use of it, and even the attempt to describe difference can happen only through further differences.

In this sense, difference is not explained by something else. It is its own presupposition, while what we can recognize are always only its effects.

Perhaps for this reason one can speak of a hierarchy of distinction.

At the beginning stands difference in itself:

|

Yet a difference never remains with itself. As soon as it is set, sides arise: left and right, inside and outside, 0 and 1, please and thank you, observer and observed.

Difference generates a first relation:

A | B

Yet its movement is by no means finished thereby. For differences in turn differ from one another. Every new difference opens further relations, which in turn can again be distinguished. What at first appeared like a simple dichotomy begins to branch:

A | B | C | D | E …

One could say that difference has no rest. It resembles less a fixed point than a flowing stream, for wherever a difference arises, it opens new possibilities of further differentiations. From a single distinction there thus grows a network of relations that unfolds further and further.

In this way, numbers finally arise as well:

0 | 1 | 2 | 3 | 4 …

Number then no longer appears as an original unit, but as the expression of an already enacted differentiation. What we count are not things in themselves, but distinguished positions within an order of relations.

Perhaps it is precisely here that we can understand why mathematics can unfold into ever more complex structures. Every new distinction opens further possibilities of distinction, so that the building grows, as it were, out of itself, without having to touch its actual origin once more.

And yet this origin remains present.

Two movements seem to take place at the same time. One leads from a single difference to ever richer networks of relations. Difference, as it were, multiplies itself and brings forth an almost unlimited diversity of possible relations.

The other movement proceeds in the opposite direction. It does not ask after ever new differences, but after their common origin. It leads diversity back to its simplest form, until finally nothing remains but the original possibility of distinguishing at all.

Perhaps both movements meet exactly there — not in A and not in B, but in that which first makes both possible at all:

|

If this is so, then before every axiom there does not stand yet another axiom.

Before every number, before every definition, and before every theory stands difference itself.

Not as a thing, not as a substance, and not as the final building block of reality, but as that original operation through which reality can appear at all as something distinguishable.

Chapter VI

The Nature of Number

We have seen above that difference forms the actual foundation of every statement. Language still shows comparatively openly that it rests on this presupposition: every word, every concept, and every meaning gains its determinacy only by differing from something else. Mathematics, by contrast, already begins with statements — and precisely for that reason it easily misses how it itself arises at all.

So let us begin with zero.

What is 0?

Is zero perhaps nothing, as the axioms of arithmetic seem to suggest?

Zero is difference.

It is not nothing, but that potential from which everything else unfolds. In this sense, it is difference of the first order: the still indeterminate possibility of a difference before concrete sides have emerged from it.

At the same time, zero forms the interface between the negative numbers — –1, –2, –3, and so on — and the world of positive numbers:

```
–1, 0, 1 → –1 | 1 → –2 | –1 | 0 | 1 | 2
```

Zero is thus the potential of difference in itself, that impossible which first allows the possible to come forward at all. Consequently, when I write 1, strictly speaking I am already writing:

0 | 1

This zero-reference remains effective as potential, as the boundary between the possible and the impossible, and as the counter-position of every determination in all subsequent numbers, operations, and self-references, even when abbreviated notation omits the sign 0, but not the relation.

And already here it becomes visible again how the order of difference lets the explanation fall apart into further poles. The 1 is what arises as soon as we understand 0 as the impossible. For 0 makes sense only if it defines itself through a counter-pole — through all that is possible, and therefore through 1.

The one, logically inferred, is the possible.

Precisely here we can observe distinction in its unfolding, for the 1 is contained in every number that is. It is what is possible, and it repeats itself in every further numerical determination:

$$0 \rightarrow 1 \rightarrow 1+1 \rightarrow 1+1+1 \rightarrow 1+1+1+1$$

If further numbers now exist, all of which carry the 1 within themselves, the difference of the third order shows itself therein. From the 1, understood as the set of everything possible, a further distinction is made. This distinction, however, is not merely an act of pure separation, but at the same time an act of relating to itself.

The 1 mirrors itself in the 2, that is, in 1 + 1, for difference cuts through a relation that already exists. The space of everything possible is divided by fanning itself out; the 1 begins to refer to itself:

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0 | 1 | 11 | 111 | 1111
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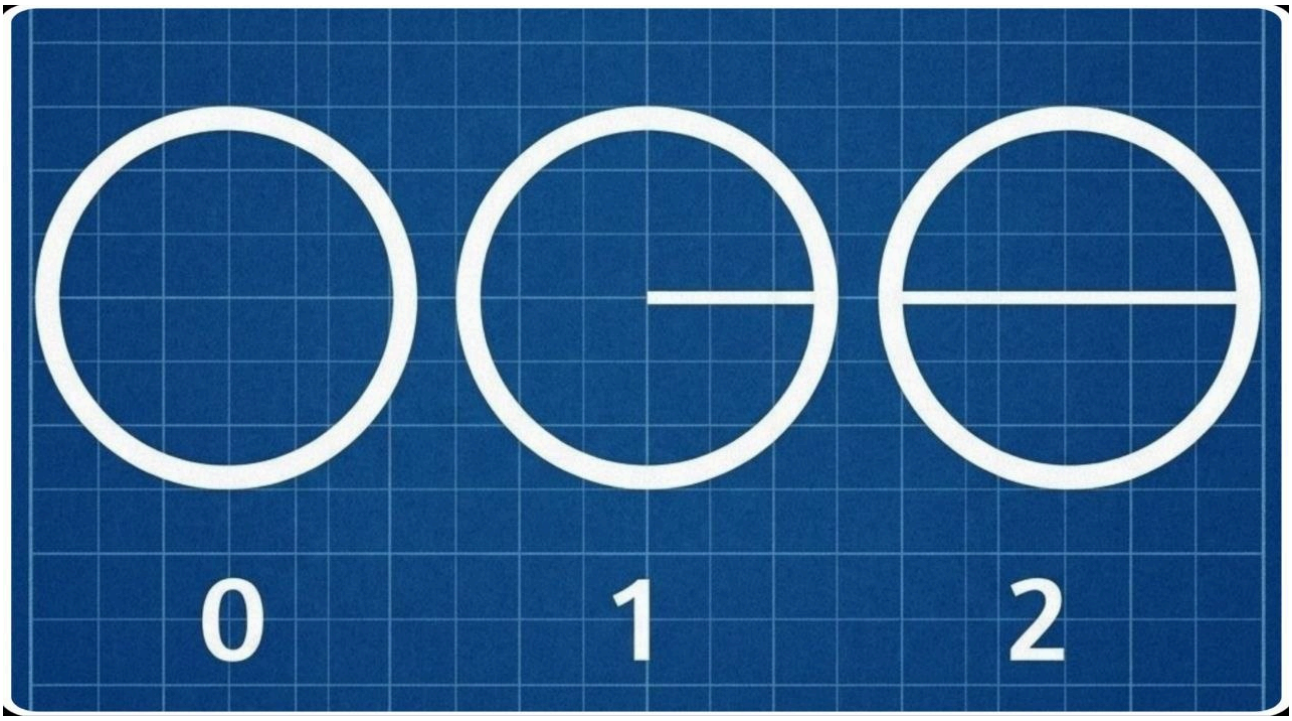
The same process can be viewed from another perspective without its effect changing.

The difference from 0 to 1 generates the third order of distinction, that is, the 2. At the same time, the representation 0 | 1, considered formally, already contains three elements. Correspondingly, 0 | 2 or 1 | 2 also imply a 3, even though it is not expressly described.

Here the relations unfold within distinction itself. Relation becomes the element of the genesis of numbers and thereby inscribes itself immediately into the form of their representation.

This form is at the same time geometrical.

The 0 is a circle: closed in itself, without beginning and end. The 1 appears as a radius within this circle, as a cutting line that leads from the center to the boundary and distinguishes the closed space for the first time. The 2 is the mirroring of this radius. From the single line there arises a diameter; the circle is now cut through at two points, and from its 360 degrees there arise two regions of 180 degrees each.



The relation of 3 breaks these 180 degrees open again and distributes them over three relations through which it describes itself as number. The triangle arises as the 180 degrees unfold relationally into

$$60° + 60° + 60°$$

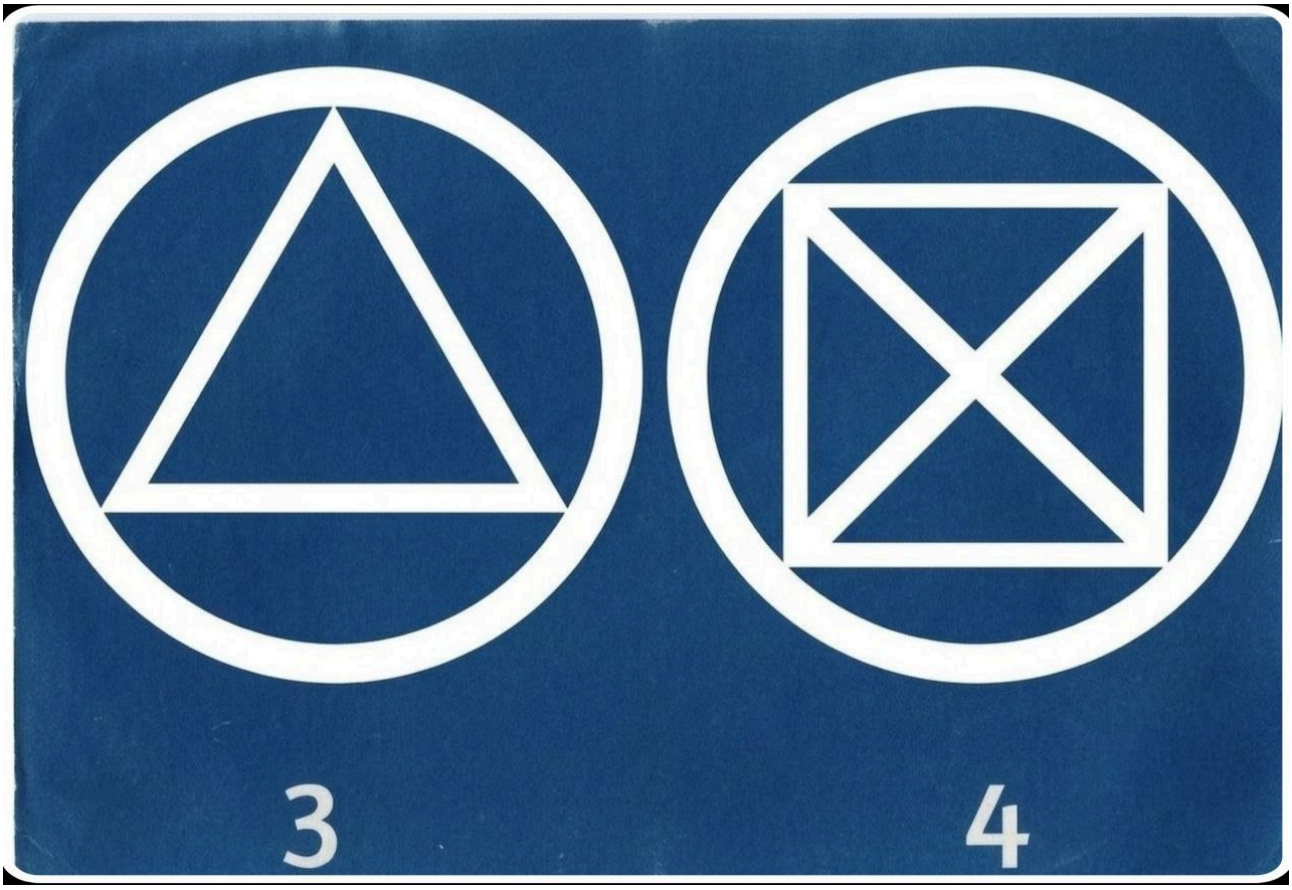
That zero has for centuries been represented as a closed circle does not appear accidental from this perspective. Its form would then not be an arbitrarily chosen sign, but the expression of its essence: it symbolizes a whole without determinate content, a boundary without concrete determination — a space of potential that has not yet unfolded.

Yet as we see, difference begins to unfold this space of relations.

0, 1, 2, and 3 are relations that are each closed in themselves. The 3 proves to be stable and once again uniform with respect to the parts of itself — similar to zero, though on a level that has already unfolded.

With the 4, therefore, something new arises, something that stands outside this basic relation from 1 to 3.

Let us again call upon geometry in order to make this transition visible. A triangle has 180 degrees, distributed over three angles. A quadrilateral, by contrast, represents a return to 360 degrees, which now show themselves in four angles of 90 degrees each.



At the same time, a quadrilateral can be divided into two triangles by drawing in a diagonal. It is therefore a combination of triangle and stroke; it arises from the relation of 3 and 1 and at the same time remains implicitly linked with the 2 — as the mirroring of the 1, as radius and diagonal.

The previous order could accordingly be read as follows: the 0 belongs to distinction of the first order, to the | itself. The 1 and the 2 manifest the second order, namely side-selection and mirroring. The 3 is the first figure of the third order, a relation of relations. The 4, finally, is the first combination of the relations from 1 to 3.

Yet precisely here a break occurs.

For the two triangles that constitute the quadrilateral, and therefore our representation of the 4, change the angles of the original triangle. Three times 60 degrees become twice 45 degrees and once 90 degrees.

Difference always distinguishes differently.

For this reason, what arises here is not a mere return to the 3 and to the original triangle, but a new relation. Yet what relation has actually arisen thereby?

Pictorially, the relation $0 \mid 3$ is a triangle within a circle. The relation $0 \mid 4$, by contrast, is a quadrilateral in the circle, divided by a diagonal into two triangles — that is, $1 \mid 3$.

In their combination this yields:

$$1|0|3|2 = 4$$

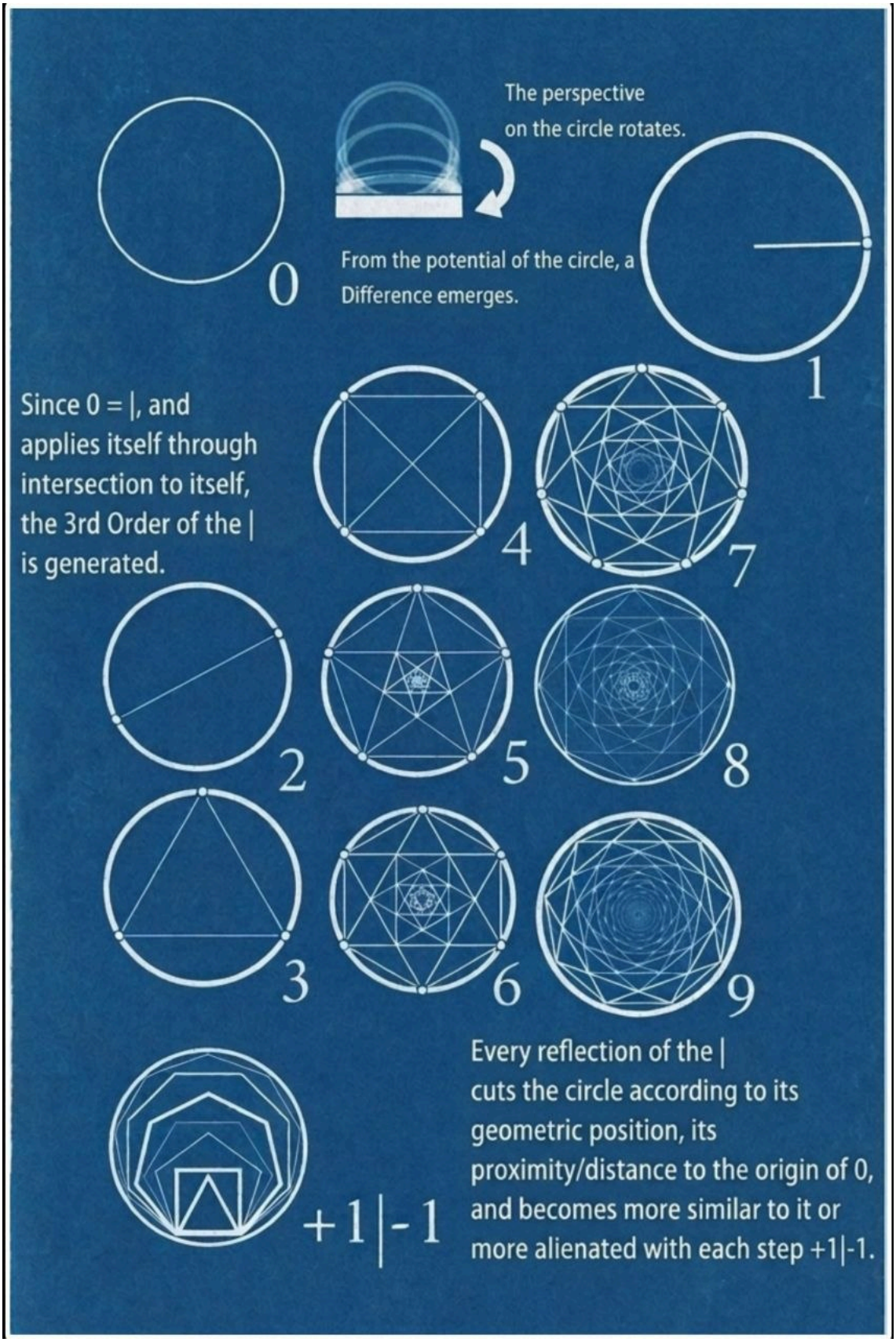
This actually produces two triangles in one deformed circle each, for the angles have shifted in the transition from 3 to 4.

The circle is zero and therefore at the same time the structure of distinction. At the 4, however, it becomes visible that the quality of difference changes and is displaced into a new relation: into the relation of pi, π.

Pi describes the ratio of circles. It does not appear here as an arbitrary constant, but as the necessary consequence of a new level of relation. As long as the structure exists exclusively within the circle, its relations unfold in its interior. As soon as it relates to the circle itself, however, a new ratio arises — that between circumference and diameter.

Within this movement, numbers can be ordered according to their ontological quality:

Region	Numbers	Character	Geometry
Primordial relation	0	Potential	Circle, closed
Dichotomy	1, 2	linear	Radius, diameter
Closed relation	3	triangular	Triangle, 3 × 60°
Combinatory relation	4+	deforming	Square, two distorted triangles
Emergence of π	from 4 onward	self-reference of the circle	Circumference-to-diameter ratio



As we can see, the number of relations increases with every advancing number. The relations that have already arisen begin to relate to one another, and with every new self-reference the fabric of their possible connections expands at the same time. Every number thereby reflects

the preceding relations, takes them into itself, and transforms them into an increasingly richer structure.

The 5 continues precisely this movement by fanning out the previous relations once more and transferring them into new relations. As a pentagon, for example, it can be understood as a quadrilateral with an attached triangle, separated from one another by a stroke. Likewise it can be viewed as a structure of three triangles separated from one another by two strokes. Five strokes, in turn, form the pentagon itself.

No matter from which perspective we look at the 5, it confirms that every new number is a relation of the preceding numbers:

$$5 = 4 + 1$$

$$5 = 3 + 2$$

$$5 = 3 + 1 + 1$$

$$5 = 2 + 2 + 1$$

$$5 = 2 + 1 + 1 + 1$$

$$5 = 1 + 1 + 1 + 1 + 1$$

This relating continues with every further number up to 9. The 9 occupies a special position here, for it is the relation of 3 to itself:

$$3 + 3 + 3$$

or

3 × 3

With this, we reach the end of our initial triangular structure. At the same time, a break again shows itself here, similar to the one that appeared earlier with pi. While pi, however, represents more of a soft break, a bending or displacement of the existing relation, the 9 forms an actual breaking-edge: the provisional end of those relations that unfold immediately out of the triangle.

The next number, 10, is indeed again a combination of the numbers 1 and 3, but now on a higher level. Self-referentiality must open this new level, for the numbers 1 to 3 form the original triangular structure, while 4 to 8 represent combinations of triangles that nevertheless do not yet go beyond the self-referentiality of 3 + 3.

Only with the 9 does a threefold self-reference come forward for the first time:

$$3 + 3 + 3$$

Every new self-reference must accordingly draw in a further level — and precisely in this way the numbers beyond 9 arise.

Our familiar notation has a peculiarity here. It expresses the transition into a new level explicitly as a return to 0, that is, to the origin of distinction. The 10 is the reversal of

0 | 1

and becomes

1 | 0.

It thereby indicates the beginning of a new level of relating. The next number can therefore not be merely a linear continuation of what came before; it must introduce a new order.

In other forms of representation, such as Roman numerals, this transition comes forward less obviously. Other representations of number also generate other breaking-edges. Thus in Babylonian counting the breaking-edge to the next level would lie at 6.

The relations themselves, however, remain untouched by this. They are invariant with respect to their particular representation.

What remain invariant are:

1. the orders of difference — first, second, and third order,

2. the forms of self-reference — simple, twofold, threefold, and so on,

3. the emergence of constants such as π, φ, or e,

4. the level-structure of basic relations, combinations, and meta-levels.

What varies, by contrast, is the concrete form of their representation:

1. the notation — decimal, Roman, Babylonian, or binary,

2. the breaking-edges — for instance at 9, at 6, or at 59,

3. the symbols used — 0 to 9, I to X, or the corresponding cuneiform signs.

Number is therefore not the mere counting of things. It is the process in which reality distinguishes itself, mirrors itself, structures itself, and finally refers to itself. Every number forms a moment within this process — and every new number a new quality of relation.

Chapter VII

The Rhythm of Number

We have now understood how numbers come into the world and that, with every further relation, these numbers establish a larger network of their self-referentialities. We have also realized that this self-referentiality is invariant — that is, that it does not make itself dependent on the form of representation. This also explains why geometry was initially introduced here tacitly, without a deep explanation. For geometry is relation in an image. A triangle within a circle was the 3 (0|3), if you remember. If the triangle is expanded by one further stroke (a 1), a quadrilateral arises, and with another stroke a pentagon. If you now think carefully, you will recognize how the emerging geometry approaches a circle more and more with every further stroke you add. What was previously a triangle in the circle is, at some point, with every further step, almost no longer distinguishable from the circle in itself. In infinity the difference disappears entirely and the circle has returned again; this makes clear that infinity is a falling-back into the potential of 0.

With every further step, then, a rhythm seems to unfold. And this is not a new thought — look back into our childhood. The act of counting, and of learning numbers as such, is a sing-song that impresses itself upon us: 1 and 2 and 3-4-5-6-7-8-9. In doing this we use the fingers and establish a relation. Each finger becomes a number and shows its connection with the others.

Counting is therefore the rhythm of distinction itself, through itself.

Yet this rhythm is invariant, as shown above. It is always the same; it is always one difference more, one further distinction of itself. It shows that the invariance of difference, in its three orders, functions always in the same beat.

What this means is that difference stabilizes itself; it can stand only in a determinate relation to itself. But what kind of relation is this between differences?

Here we return to the breaking-edge between 3 and 4 and to the emergence of pi. Up to the 4 we move in relations of 1 (1 | 1+1 (2) | 1+1+1 (3) = triangle); with the 4, however, the level of combinatorics changes, for from now on it is no longer merely a combination of 1, but a combination of 1 and 3 (2× triangle = quadrilateral). All numbers after 3 are only combinations of 1 and 3 (for example, 7 = 3+3+1).

Thus every structure of numbers beyond the basic structure of 3 possesses both inner stability and a relation to the circle. Every difference changes the geometry so that it comes closer to this circle.

If we call to mind that every circle has 360°, then every step of counting must be a segmentation that stands relationally to this circle, for every number, in its geometrical form, approaches it.

Yet between 1 through 4 and the following 5, the break of a new level occurs, one that the circle brings with it. Potential brings relation, as we have seen, and this relation always distinguishes differently.

Previously, after all, we had always seen symmetry arising out of the act of asymmetry.

The 1 is a radius that cuts the circle, I said above. This is the act of asymmetry through the symmetry of the line, which breaks the perfect symmetry of the circle and unfolds the potential. The symmetrical line mirrors itself as 2 and becomes self-referentially more symmetrical with every distinction. The 3 forms itself as the triangle of this symmetry into a further, more stable form of self-referentiality, which is likewise symmetrical, for every interior angle is uniformly 60°.

The 4 is symmetrical again, even though it is a combination after the soft breaking-edge of the transition to pi. The quadrilateral can be divided symmetrically, and its parts are likewise symmetries.

With the number 5, however, this symmetry of the numbers 1, 2, 3, and 4 breaks. Let us first look at the 4, the quadrilateral. The four corners of the quadrilateral cut the circle into four segments of 90° each; a pentagon, by contrast, cuts the circle five times and has thereby created five segments of 72° each. The figure of the quadrilateral therefore expands and displaces the points of intersection with the circle. The interior angles likewise change from 4×90° to 5×72°. Now, however, comes the decisive point. The 4 implicitly carried the 2 within itself and was symmetrical. The 5 also carries this symmetry within itself, but must now redistribute it within the relation that conditions the 5. The 4 could be divided into two triangles with 90° + 45° + 45°, and precisely this symmetry of the 180° must now redistribute itself over five angles.

If we divide 5, the result is 2.25, and precisely this is the offset that the triangle must have in order, within the pentagon, to appropriate the symmetry of the two triangles of the quadrilateral; for now 5×36° is exactly 180°, and a new kind of symmetry begins. For every 45° angle becomes 36°, that is, it is reduced by exactly 9°, or by one breaking-edge; and if we distribute these 9° over the four angles of the quadrilateral, the offset is exactly 2.25°.

Why 2.25? The angular offset is 2.25, and likewise 2+2+5 = 9 — precisely the 9 degrees of offset needed to restore symmetry from 4 to 5. The breaking-edge of the 9 already implies here the change of level.

Where in the world do we see precisely this rotation of geometry, if these relations appear fundamentally in the relation of numbers?

If one looks closely, one recognizes that every number of the Fibonacci sequence follows this principle, which triggers a minimal but steady rotation of the geometrical order. It is precisely those 2.25° that are necessary in order to realign symmetry on a higher level. Fibonacci is therefore not merely a sequence of numbers, but the expression of a recurring self-rotation of difference: that of a growing triangle that turns slightly further at every step.

The diagonal divided by the side of a pentagon is the golden ratio of 1.618033. It is therefore a cut through the circle and the pentagon at the same time. 360° divide into 180° and into our 5×36°.

The diagonals generate a pentagram within the pentagon, with angles that are now all 36°.

The symmetry of self-referential mirroring therefore lifts itself once again onto a higher level with the 5, for now there is growth through the levels. And this growth is self-referential through difference, which shows itself in the geometry of the pentagram. For within the pentagram lies a pentagon.

What shows itself is difference of the third order as it becomes increasingly complex in its relations. As mentioned above, it fans itself out. But the 5 has shown us the beat in which self-referentiality becomes fractal: namely, in exactly 2.25°. And thus difference stands to itself in a coherent relation only when it is changed by 2.25°. The breaking-edge of the addition of 2.25 is indeed 9; the break and the transition show themselves here in the relation of 2.25 and the golden ratio itself. For 360°/2.25° is 160°, while the golden ratio reaches these 160° in 10 steps, that is, 2×5. 1.6 × 10 is 16, the tenth part of 160°, while 18033 means nothing less than 9|3 (for 1+8 is 9, and 0 is the distinction that generates 3×3, that is, 9, or indicates 9/3 = 3), and thus unlocks the relation of the breaking-edge once again and leads it back into the ratio. It also shows that the rotation of every Fibonacci step turns by approximately 137.5° (the golden angle).

And: 360° − 137.5° = 222.5° … = 100 × 2.25°, the tenfold of the tenfold step.

It becomes apparent that 2.25 is a relation that also occurs in the triangle. 2 stands in relation to two relations, namely 1 and 3; thus 2, being between 1 and 3, is itself a relation, for 2 + ¼ (0.25) = 9/4. The 2 thereby already encodes what the 5 repeats.

```
Δ = 2.25° within the | geometry
```

The decimal places of the golden ratio, 18033, also imply the transition to the number that comes next, for 9 as the breaking-edge of the 5 implies the 6: because 1+8 = 9 and 9|3+3, that is, 9|6. The relation to the next number is thereby already fixed.

The 6 is the return into symmetry. The hexagon is the relation of the 3 to itself. If one looks at a hexagon with knowledge of self-referentiality, one sees that a hexagram can be constructed from 60° triangles. This shows self-referentiality with respect to inner substance. Yet we already know about the inner relations of every number. A 6 is likewise 4+2, 2+2+2, 3+1+1+1, 3+3, 3×2. All are symmetrical mirrorings of the basic relation of the triangle, whose total relation stands to itself in the basic beat of Δ = 2.25°.

The connection to the golden section is also carried into the relation here. To invoke once more the number 1.618033 of this section: 1+6+1+8+0+3+3 = 22 = 4, which is a part of the 6, for 2+4 = 6. Likewise, 1+8+0+3+3 = 15 = 6. Here 6+4 yields the relation 1|0. Symmetry carries itself into the next level.

Again the structure of distinction gains complexity with every number. In doing so, every new expression contains the preceding one, but generates an expansion and transformation with every expanding Δ.

The next Δ step leads us to the symmetrical mirroring of the 5. For the 7 is 5+2 (5|2); thus the qualities of these two ratios are part of the 7. It is fractal, not closed like 5, but on a new level — one that cannot quite dissolve itself if one views 7 as the relation of 3|4. The 7 is therefore a relation in which the breaking-edge π shows itself, but in variance, for it is located in a new relation, that of the 7. Pi as relation has been known for a long time; today one says that it is an approximation to π = 22/7. Through the | difference we see immediately: 2+2|7 = 4|7 = 4+7 = 11 (22/2) = 1+1 = 2. This tells us that here we see a mirroring of what the relation π is between 3 and 4.

If we draw a circle and wish to approximate its circumference not by an irrational number but by a fraction, the classical choice is 22/7.

Geometrically, this consequently means that any diameter fits 7 times into the circumference, approximately 22 units of length.

If we divide a circle into 7 equal sectors, we obtain a “heptagonal” structure that has been connected with the approximation to π since antiquity.

The interior angle of a regular heptagon is approximately 128.571°.

7 × 128.571° = 900° (9|0|0), which leads beyond the 360° (1|0) of the circle. It does this by a measure of 900/360 = 5/2 = 2.5. This 2.5, or 5/2, is not merely half of 5 or its relation. If we add, as above, 5|2 = 5+2 = 7, one sees again the self-referentiality in the nature of numbers, because they are an expression of the |.

Beyond this, the following reference can also be discovered here: 2.5 is the ratio of 5|2, and applied to itself it shows itself as 2|2|5, or Δ = 2.25°.

The 8 shows us an inner return into symmetry. It smooths the fractal eruption of the 7 by appropriating its relations. Let us look at the inner symmetries of the 8:

$$\begin{array}{lll} 8/2=4 & 8+2=10=1 & 8-2=6 \\ 8/4=2 & 8+4=12=3 & 8-4=4 \\ 8/8=1 & 8+8=16=1+6=7 & 8-8=0 \end{array}$$

What we see are mirroring symmetries in 4, 2, 1 and the asymmetry 1, 3, 7 in one figure, perfectly distributed and closed in upon itself, with the diagonal mirroring of the 4 as its rotational point.

If we look at the graphic of the 8, in which the preceding patterns repeat, we see a symmetrical distribution of the triangles; the morphological self-repetition here is not fractally open as with 5, 6, and 7, but is to be located in the symmetry of self-relation.

Let us now come to the 9. Like the 8, it is highly symmetrical (8+1 = 9), but the 5 and the 4 are its greatest relations (5+4 = 9), which is why it is fractal like the 5, but also a breaking-edge like the 4, with respect to the 7, which appears as 9 = 7+2 = 7|2 = 7|1|1. This makes the 9 into a mirroring of the 7 from out of the basic structure of the 2.

It therefore seems that the 9 holds several special properties ready for us. Let us look at it more closely:

$$\begin{array}{ll} 9+1=10=1|0 & \textbf{the next level of numerical relations and again the first number, 1} \\ 9+2=11=1|1=1+1=2 & \\ 9+3=12=1|2=1+2=3 & \\ 9+4=13=1|3=1+3=4 & \end{array}$$

The 9 therefore generates a recurrence of the basic schema of numbers. It generates the new relation out of itself. But this works only if the 9 stands in connection with the 0. We have seen that the 0 is a circle. This circle has 360°, and precisely here sits the relation between 9 and 0, for

360 = 3+6+0 = 9|0; the relation shows itself. Now we understand why the numbers 0–9 form a self-referential structure.

$$
\begin{aligned}
&9 \times 9 = 81 = 8 + 1 = 9 \\
&9 \times 2 = 18 = 1 + 8 = 9 \\
&9 \times 3 = 27 = 2 + 7 = 9 \\
&9 \times 6 = 36 = 3 + 6 = 9 \\
&9/4 = 2.25 \\
&9 \times 2.25 = 20.25 = 2 + 2 + 5 = 9 \\
&9/5 = 1.8 = 1 + 8 = 9 \\
&9/2 = 4.5 = 4 + 5 = 9 \\
&1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45 = 4 + 5 = 9
\end{aligned}
$$

The 9 shows itself as the self-reference that generates and transforms at the same time. It is the keystone in the system of numbers. From here onward, what unfolds are only other kinds of relations under this basic relation of the preceding numbers.

What we have seen in this chapter is a systematic unfolding of difference itself, |, in relation to itself. From a fundamental dichotomy A|B → 0|1, we have seen that the basic configuration 1|2|3 arises and that it finds its equivalence in geometry, in that of the triangle. Across the numbers from 4 to 9 we have always applied the same principles in order to see that they contain the connections of their preceding relation. Everything culminates in the relation of the 9. In the next chapter we will consider the relation of difference itself and recognize what arithmetic is trying to achieve at all, and where its axioms fail.

Chapter VIII

The Systematics of Distinction

```
Difference of the first order:
|
```

This difference generates a dichotomy, a relation.

```
Difference of the second order:
0|1⇒ (0 ≠ 1)
```

And difference always distinguishes itself differently.

```
This is difference of the third order:
0|1|2|3|4|5|6|7|8|9
```

These were the relations already introduced above, which difference generates in reference to itself. The different orders are represented here only hierarchically, for this differentiation is not to be understood temporally in its relations. This view of the relations of numbers to one another, in this sense, elevates difference in its function. For | thereby seems generally to carry a kind of decision, which first creates the relation in the same sequence again and again.

In the preceding chapters we have seen the generative difference |. It differentiates and relates fundamentally identical things from the same set: A from B from the set A to Z. It conditions the set that it distinguishes. With numbers this can be shown well: 0|1 is the condition for 1|0 and conversely; after the second order, the third follows with 0|1|2|3|4…|9.

But what showed itself was an increasing complexity of connections. From the assumption 0|1 there unfolded such a dense relation that it culminated in 9.

We first considered this relation in the rhythm of number itself.

Every number is one difference away, and the smallest possible difference of the geometry of numbers was Δ = 2.25°.

Thus difference is quantified as Δ. It follows logically:

1Δ2Δ3Δ4Δ5Δ6Δ7Δ8Δ9

Thus difference is Δ in geometrical relation and | in abstract, linguistic relation A|B. But how can this be explained? For apparently difference does not possess only one role that it takes on.

In the preceding examples, I deliberately refrained from explaining why, for instance, 1.8 may be reduced to the digit sum 9. And if you have held out this far, the penny will now drop, and in a few paragraphs you will understand that this is not Kabbalah number-mysticism, but a self-generative, invariant prism of its self-relation.

$$Whatare + -/\times\sqrt{\div} = and.?$$

These are notations in mathematics. They say which process is being described.

They say how numbers stand in relation to one another.

In arithmetic this happens in a beat:

$$1+1+1+1+1+1+1+1+1=9$$

But we know that this means:

1Δ1Δ1Δ1…

Thus | is also + or -, for that is always this procedure of the path.

6/2 = 3 — this mathematical question is like a relational specification in a network of relations. It shows itself as (6|2)|(0|3), where (6|2) acts like A and (0|3) like B, for A|B. What we are looking at is always a nested relation of itself. Fundamentally, therefore, we are asking only questions of a determinate depth-relation within 0–9. The operators used, such as + - × ÷, are only the way in which we look at the network of reflection. | is the general view; + is the rhythmically expanding beat +1 $+\Delta$, while - contracts the relations in the same beat.

Modes of difference:

```
|+    Expansion (addition)

|-    Contraction (subtraction)

|×    Resonance (multiplication)

|÷    Unweaving (division)

|^    Exponentiation

|√    Root

|=    Equilibrium (locking-in)

|.    Scale-flip (decimal point)
```

In order now to clarify why the digit sum of 1.8 is valid: in our new understanding, the . does not appear as an irrational number-combination because the number is not whole, but as a flip of the level; for from potential, difference arises, which at 9 climbs onto a new scale. In the opposite direction there is also a difference | — the scale-flip (decimal point). It is a way of locating relations without solution on another level and of limiting the relation, without dissolution, in self-reference.

1.8 = 1+8 = 9

$$1|.8|=1|+8|=0|9$$

$$1|8|1|8|0|9$$

(1|8)|(0|9) — this is the relation we are actually considering, but through our lenses of + and . as well as =

And logically, in the relation, 1+8 = 9 shows itself.

Digit-sum reduction:

$$47 \to 4+7 = 11 \to 1+1 = 2$$
$$13 \to 1+3 = 4$$
$$2 \times 4 = 8$$

Consequently, 47 × 13 should have the digit sum 8.

Check:

$$47 \times 13 = 611$$
$$6 + 1 + 1 = 8$$

This works in all | relations.

The number pi consequently appears as scale-flip between two levels: the circle as well as the polygon, the discrete and the continuous. The connection of the two preserves the invariance of the information that | generates in its self-structure. That π cannot be written out is the result of the depth within the relations into which pi is embedded, for it is every relation with which it stands in relation. The question of which relation exactly we are seeing is answered by the viewing angle of difference, whether $|_+$ or $|_-$, and so on.

The invariance of what a relation is lies in the dichotomy itself, for information is the distinction of everything from something, and something always has two poles so that it can stand in a relation. Information is therefore distinction, and its relation to us is the operator.

In every invariant system there must be a connection that serves as reference to itself. If pi is the relation of the geometry of information, what could be equivalent to this in the relation of numbers?

This relation would have to be self-referential to all numbers and, in its ratio, mirror the relation of numbers itself.

39.19 is precisely this number.

(3|9) | (1|9) is full of meaning; draw together here once more the insights of the preceding chapters

3 to 9 is the stability of the first relation 1|2|3 and therefore of the triangle, up to its last stable form of self-referentiality, the 9. The . is a scale-flip and connects 1 to 9 in the sub-level of the relation of 0–9.

3+9 | 1+9 = 12 | 10 = 3 | 1 — the basic relation with 2 as relation, implied in |

$$3 \times 9 | 1 \times 9 = 27 | 9 = 2 + 7 | 9 = 9 | 9 = 1$$

Thus it contains the basic relations 1-2-3 as well as the breaking-edge of both scale-levels.

$$39 + 19 = 58 = 5 + 8 = 13 = 1 + 3 = 4$$

The 39.19 apparently extends beyond the breaking-edge of the ¾ relation.

$$39 - 19 = 20 = 2 + / - 0 = 2$$

The reduction of the ratio likewise mirrors the expansion through addition.

$$39 \times 19 = 741 = 7 + 4 + 1 = 12 = 1 + 2 = 3$$

$$3 \times 9 \times 1 \times 9 = 243 = 2 + 4 + 3 = 9$$

$$360^\circ / 39.19 = 9.186016841 = (9|8) = \Delta 2.25^\circ, where 9/8 = 1.125 (½\Delta)$$

Thereby 39.19 shows, in its projection onto the circle, its own relation as Δ across two levels, indicated by |.

(3+9) - (1+9) = 2, keeping the scale-flip . in mind, yields (1|1), and thus (1|0|1) if one inserts the scale-dependent relation again. This is like the invariant breath of the system itself:

Arising > Passing-away > Arising

This shows:

1. Number is a position in the space of distinction.

2. Operators are modes of difference.

3. Arithmetic is the self-reflection of this difference.

4. Information is nothing other than distinction.

5. Invariance arises when a difference becomes referential to itself.

Chapter IX

The Calculation, Please!

We have now found a new foundation, based on which we have seen the unfolding of numbers in their relations as geometries.

Nevertheless, we saw that operations such as addition, subtraction, and so on are modes of difference.

Now we turn to the consequences of these ideas and will look at what this means for numbers and for the understanding of them.

Let us take the following example:

$$729 + 184 = 913$$

Let us write this out once, broken down hierarchically:

(7|0|0) |₊ (2|0) |₊(0|9)

|₊

(1|0|0) |₊ (8|0) |₊ (0|4)

|=

(9|0|0) |₊ (1|0) |₊ (0|3)

Thus, in fact, the relation of this task presents itself when written out.

The hundreds, tens, and ones places are implicitly already in the notation.

(7|2|9) |₊ (1|8|4) |= (9|1|3)

Each direction has three stages of iteration; in our example it is expansion:

Expansion:

```
|₊  Stage 1: Minimal beat (A|₊1)
|ₓ  Stage 2: Iterated expansion (A|₊A|₊A...)
|^  Stage 3: Iterated iteration (A|ₓA|ₓA...)
```

Contraction:

```
|₋  Stage 1: Minimal back-beat (A|₋1)
|÷  Stage 2: How often does B fit into A?
|√  Stage 3: Which A, iterated, gives B?
```

The carry that arises at every crossing of 9 becomes a feedback into the superordinate level. Consequently, the relationing + is an expanding relation of the sequence of the hierarchy of the considered relationship of the numbers and their positions.

Now we understand why:

$$47\times13 \to (4+7)\times(1+3) \to 2\times4 \to 8611 \to 6+1+1 \to 8$$

The operator |. and the digit sum are scale-spanning operators

The digit sum is a scale-collapse:

(4|7) |. → 4 |₊ 7 → (1|1) → 2
(1|3) |. → 1 |₊ 3 → (0|4) → 4

$$2|_{\times}4 = 8$$

Let us look at the following operation: 7×3 = 21. Here, in our notation, it says:

```
(7|×3) |= (2|1)  or (7|+7|+7) |= (2|0|1) This shows us how the operator works as a
relation in the network.
```

In conclusion, the following insight likewise results: The 3rd-order difference acts on itself, for:

The basic difference is |+ or |–, which is applied to itself because it is a self-referential distinction +|-. The self-referentiality of the distinction itself leads us to the next chapter.

Chapter X

The Negative Number and Its Meaning

Before we look at what exactly the negative number means, we must recognize how relational expressions themselves are formed.

Let us again make use of the geometric relation of π:

$$22/7 = 3.142857143$$

```
((2|0|2|) |÷ (0|7)) |=  (3|.1|4|2|8|5|7|1|4|3)
```

We change the perspective:

$$((2+2)+(0+7)) = ((3+(1+4+2+8+5+7+1+4+3))$$

This yields 2=2, by which we see that the expressions are equivalent.

Both describe the same relation in other contexts. Initially it is a division, that is, the relation of 22 around its 7th part. This relation is based on the induced viewing angle ÷ (|÷). From the outside it shows two equivalent notations; at the core, however, two different but equivalent paths to the relation appear.

This relation, as we understood it in the last chapter, says +|-. In our example 22/7, this path is negative, thus -Δ, for the operator / assigns this viewing angle.

In the preceding text of the tractate we have always concerned ourselves with positive relations, the left side of +|-. Now, however, it is revealed that the contraction of the polygonal number structure contains a mirror l, for = is |= and that is merely one perspective of this relationship + = -.

Thus the negative is the relation of the positive, an unfolding of our distinction +|- +Δ|-Δ. Applied to itself, the mirroring arises, which again refers to the basic relation -×- (minus times minus) = -|- → +|-. This relation is the self-referentiality of Δ.

```
-2+-4 = -6   →    (-Δ2) + (-Δ4) =  (-6Δ)
-2×-2 = 4    →    (-2Δ) × (-2Δ) = (-×- (4)) = 4
```

It becomes apparent that the negative number is not only another relation; it is another viewing angle on the same relation. It is equivalent to positive number-space, but it does not describe the things that arise from relation, rather the path there, relative to the source, the 0.

```
(-1|0|1)
```

At last, a real explanation for the behavior of √ also seems to have been found.

```
√16 = 4    but this is a convention, for every root likewise yields a negative value.

√16 = ±4   one does not consider -4 a solution to the question of which x satisfies
x² = 16.
```

But in the face of the insight we gained before, a self-referential distinction appears.

```
|√(1|0|6) |=  4Δ in order to close the relation 1|0|6 → 10 to 6 = Δ4
```

But -Δ4 likewise holds in the reversed relation 10 → 6.

Plus and minus are thus two viewing angles on their origin. - looks from the outside into the relation, while + is the view out of the expansion.

The case √-1 consequently needs no imaginary numbers for explanation, but only an expansion of its inner relation in order to recognize why it is possible:

√-1 corresponds to itself as relation to itself. For this we use the general difference for the proof:

```
−1 | +1  this corresponds to 0, for -|+ and 1|1 are both self-referential |, that is,
one and the same difference.
```

Here this means -1 is the mirroring of +1 across the 0, for | is 0; consequently ...-1|0|1... the 3rd-order difference, if we relate it to itself.

Every 1 is the beat of arithmetic, that is, Δ.

We thereby conclude:

+1|-1 = -1|0|1 = -Δ|+Δ = +|- = -|- this is the compulsion to distinguish itself

-|- becomes +|- and the chain repeats itself.

Instead of an imaginary number for the explanation of √-1, we use the geometry of the difference itself Δ=2.25°.

```
-Δ|+Δ   =   -2.25° | 2.25° =
```

-2.25 + 2.25 = 0 -2.25 - 2.25 = -4,5 = -4|5 -2.25 × 2.25 = -5,0625 = ((-5) |. (6+2+5)) = -5|4 -2.25 ÷ 2.25 = -1 Minus one therefore means -Δ = 0 - 1 = -1 and the preceding numbers also follow this logic

$$-4|5 = -4\,\Delta\,|5 = 1 - 5|4 = -5\,\Delta\,|4 = -1$$

√-1 is consequently the inversion of the difference to itself, which causes a circular oscillation of its relation.

Thus √-1 is simply a wrongly posed question, for the 3rd-order difference of the relation √ can deliver no result at -Δ (-1), because no 3rd-order relation has yet unfolded in the viewing angle of the question. $x^2 = -1$ has no solution in classical mathematics. This, however, is thought without the complementarity of geometry. For as we can see in the graphic of every number, all polynomials have a geometric correspondence:

```
Polynomial       Geometric solution

x¹ = 1              One point (1 itself)
x² = 1        Two points (±1) = diameter
x³ = 1            Three points = triangle
x⁴ = 1            Four points = square
x⁵ = 1           Five points = pentagon
x⁶ = 1          Six points = hexagon
```

...

We also recognize that $x^2 = -1$ is a solution of a geometric nature, for -1 is the point at 180°. It is the relation of 2. For -1 is the relation to 1 at the same Δ distance as 2 to 0. What we are really doing here is looking at the 180° difference inverted.

It is the view of the same structure, only from outside the circle.

Accordingly, the complex plane of numbers becomes a map of unfinished relationings of Δ, by rotating this in the process:

0° → +1 (expansion),

180° → −1 (contraction),

90° → √−1 (rotation in the process = oscillation)

```
0  |  1  |  2  |  3  |  4  |  5  |  6  |  7  |  8  |  9  +Δ
                                                | -1  | -2  | -3  | -4  | -5  | -6 |
-7  | -8  | -9  -Δ
```

This represents the rotation from + to -; in the number-relationship between them, the imaginary numbers are found as intermediate viewing angles of the geometry.

```
0  |  1  |  2  |  3  |  4  |  5  |  6  |  7  |  8  |  9  +Δ
                                        -     -     -     -     -     -     -
-     -    90°
```

```
                                                    | -1  | -2  | -3  | -4  | -5  | -6 |
-7  | -8  | -9  -Δ
```

Where does this intermediate step come from, and why is it a geometric consequence of the distinction?

The imaginary numbers are perspectives between the perspectives of -Δ and +Δ

+Δ: Expansion (0°)

i·Δ: Oscillation "upward" (90°)

-Δ: Contraction (180°)

-i·Δ: Oscillation "downward" (270°)

Every complex number is a rotation

```
z = a + bi
```

In our notation:

```
z = a·(+Δ) + b·(i·Δ)
z = a·(0°) + b·(90°)
```

a: Component in the direction +Δ/-Δ (real)

b: Component orthogonal to it (imaginary)

Multiplication as rotation

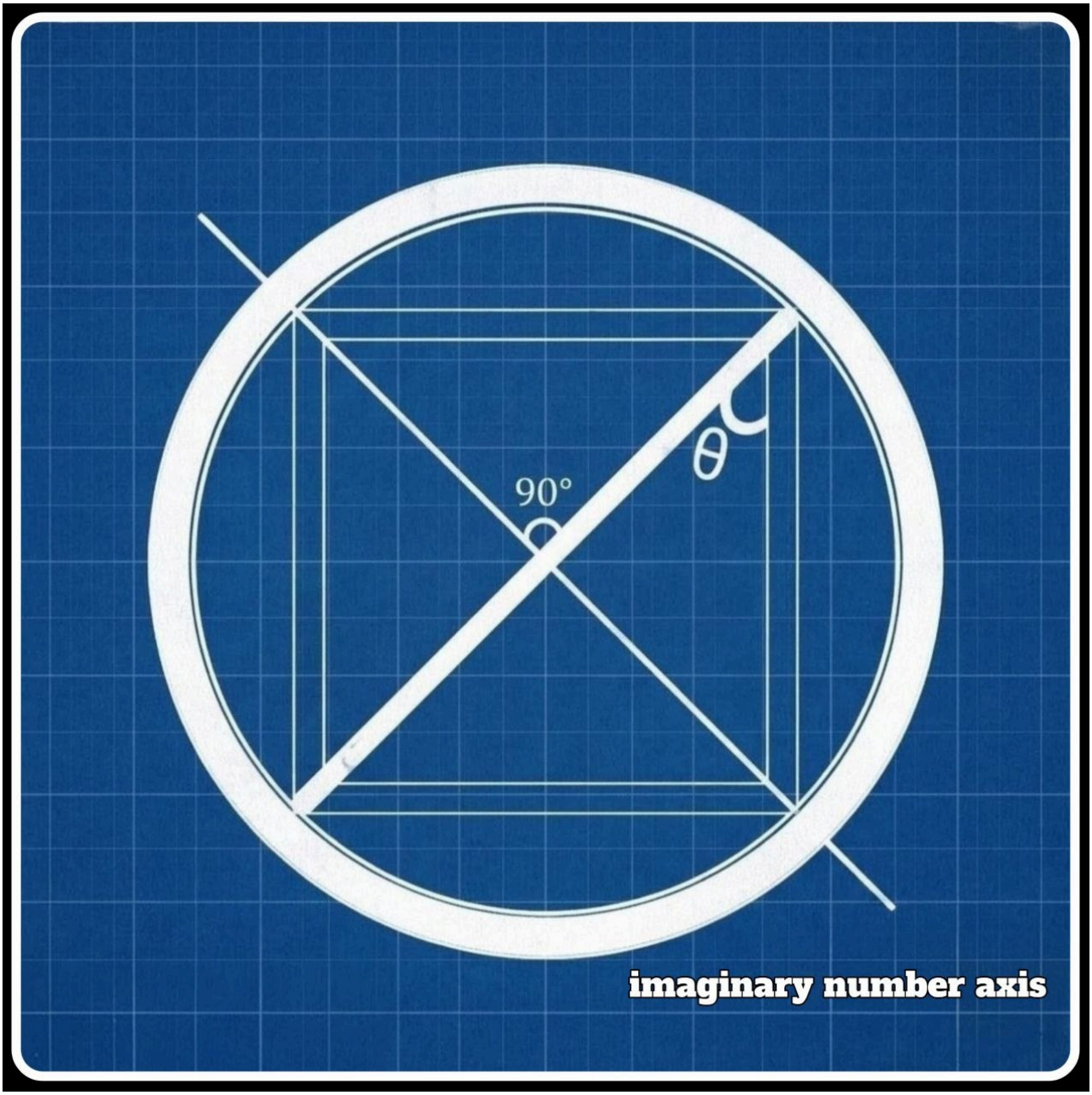
Why is $i^2 = -1$?

$$i \times i = 90^\circ + 90^\circ = 180^\circ = -1$$

Through our axiomatics of the three orders of |, it follows:

$$i \cdot \Delta \mid_\times i \cdot \Delta = (90°)^2 = 180° = -\Delta = -1$$

Thus multiplication is not "repetition," but rotation by the angle.



ei(θ) =cos(θ) + i × sin(θ) The Euler formula thereby becomes a relationship of the numbers 3 | 4. Sin and cos are to be replaced by their relations. For sin & cos come from a non-complementary system of point-/entity-thinking. Sine/cosine would here only describe how a point changes its coordinates – but they cannot express what is really happening here:

Namely, that a relationship mirrors itself orthogonally and thereby generates a new dimension of itself.

Let us remember the representation of 4: A circle with a quadrilateral divided along the diagonal. Thus 2 triangles form, which cut the circle at 4 points. The Euler formula describes exactly this ratio as 2 triangles lying rotated by 90°, in our deformed triangle of 3.

sin(α) = opposite cathetus ÷ hypotenuse = a | c

cos(α) = adjacent cathetus ÷ hypotenuse = b | c

ei(θ) =(b | c)(θ) + i × (a | c)(θ)

The geometry of 4 that I previously showed shows 4 triangles, due to the mirroring of the separating diagonal. This mirrored diagonal is the imaginary number axis. Imaginary numbers are ultimately the moment in which these two directions pass into one another, thus the pivot point of the difference.

The number is thereby not imaginary in the sense of a number outside reality, but a geometric necessity of the relation itself.

Chapter XI

Why Infinity Is Not Infinity

Infinities are not a thing of experienceable reality; they are reflections of the incompleteness of a mathematics that begins with postulates without a foundation. We begin with difference and have seen that it always distinguishes itself. Consequently, infinity is a designation for a not-complete reflection and self-referentiality whose operator is incomplete.

Infinity is not:

$$x \to \infty$$

Infinity is the non-closed question of the distinction:

$$x \to f(x) \to f(f(x)) \to \dots$$

$$\begin{aligned} 1/1 &= 1 \\ 1/2 &= 0{,}5 \\ 1/4 &= 0{,}25 \\ 1/8 &= 0{,}125 \\ &\dots \end{aligned}$$

The relation (1 | x) rotates itself in the direction of 0. It never "reaches" 0 because 0 is the potential, not a point.

The limit describes this rotation, not an "approach"

Thus the limit is not an approach to a distant point, but a closed back-referentiality without closure. Growth is not an external dynamic and not an “increase in things,” but the expansion of the relation in the distinction to itself. It does not take place linearly, but as a steady self-rotation of this relation in the golden angle – the elementary Δ-angle of 2.25°, from which all scaling processes (such as Fibonacci, or exponential growth) emerge. Infinity is therefore the question of growth, for this is the self-relation of the question.

Growth is not linear

Arithmetic:

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow ..$$

(Each step adds "one unit")

Geometry:

$$1 \rightarrow 1\Delta \rightarrow 1\Delta^2 \rightarrow 1\Delta^3 \rightarrow ..$$

(Each step rotates the relation by Δ = 2.25°)

Fibonacci growth thus means that every number is the sum of the two previous ones

That is self-referentiality: Fn = F(n-1) + F(n-2) The relation feeds back with itself.

Exponential growth y = e^x means the derivative is the process of itself: dy/dx = e^x

The function is consequently its own change.

The golden angle (137.5°) results from the relation of the process.

Fibonacci φ = 1.618… applied to the circle 360° / $\varphi^2 \approx 137.5°$

That is the optimal rotation for maximum packing and it is a multiple of Δ = 2.25°.

Chapter XII

Differential and Integral

In the classical understanding, the differential is a measure for an infinitesimally small change. One says that a function changes “by a tiny piece” and describes this piece with a limit that goes toward zero. But we have already recognized that neither infinity nor infinitely small quantities have a place in a consistent foundation. They are artifacts of an incomplete self-referentiality.

If we now acknowledge difference as the origin of all mathematical operations, the concept of the differential also changes. A change is not an arbitrarily chosen, infinitely small piece, but a movement of the relationship to itself. This movement is not linear, but fractal, for it occurs in the smallest, self-repeating steps of distinction. We have already become acquainted with this minimal movement: the Δ-angle of 2.25°, which functions as the smallest measure of the self-rotation of a relation.

A differential is therefore nothing other than the smallest recognizable self-distinction of a number or a function with respect to itself. It is not the limit value of a change, but the elementary transformation that arises when a relation completes a further step of its own unfolding.

One could say: A differential is the beat in which a relation turns its own structure onward. It is the elementary movement of the “not-yet-separated” into the next state of distinction. Every further “derivative” is nothing other than a repeated self-referentiality of this movement – an

iteration of the difference that takes place on ever finer scales, yet never becomes “infinitesimal.” It always remains a real, finite movement within the relation.

Thus a derivative is no longer an abstract “slope,” but an expression of the speed of self-rotation. It describes how quickly a relation changes its inner state when it further distinguishes itself. And an integral, in this new understanding, is not the area under a curve, but the summed movement of self-referentiality, this entire trace that a relation leaves behind when it unfolds over many Δ-steps.

Here a fundamental shift takes place: analysis is no longer a mathematics of infinitely small changes, but a geometry of self-similarity. Every change is a step of self-referentiality; every summation is a path through the space of these distinctions. “Infinitesimal calculus” becomes fractal calculus, a description of the continuous, but never infinite, self-movement of difference.

This makes clear that differential and integral are not isolated tools, but natural consequences of the fundamental operation of |. They are the language in which the self-referentiality of difference expresses itself, namely not as an approach to an infinite goal, but as processual movement within a finite, self-repeating system.

Standard Definition of the Differential

$$f'(x) = lim(h \to 0)[f(x+h) - f(x)]/h$$

Problems:

What is h? (A number? A symbol?)

Why does it become "arbitrarily small"?

What does the limit "really" mean?

Geometry:

f'x = Rate of the Δ-rotation of f at position x

fx is a relation (not a "height").

f'x describes how quickly this relation rotates by Δ = 2.25°.

Higher derivatives (f'', f''', ...) are iterated self-referentialities of this rotation.

Standard Definition of the Integral

∫ fx dx = Area under the curve

Problems:

Why is "area" relevant?

What does "under a curve" mean?

Why is this the inverse of the derivative?

Geometry:

$$\int f(x)dx = Accumulation of all\ \Delta - rotations of f$$

The integral "collects" all rotation steps.

It is the total movement of the relation through its space.

The "area" is only a 2D projection of this movement.

New Geometric Symbols of Distinction

|Δ is the smallest self-rotation, differential

|Σ|Δ is the accumulated self-movement of Δ, the integral

Chapter XIII

Is That Still Probable?

In the face of this extensive exposition of the interconnections, one last bastion of mathematics presses into focus. The edifice of thought of probability created by Laplace. This idea is the short circuit that necessarily springs from the incompleteness of axiomatic arithmetic. If one does not recognize one's own foundation, then one moves in a fog of possibilities.

```
A simple example  3|x|5
```

Which number is being sought is, in probability mathematics, the manifestation of ignorance about Δ. It forces arithmetic into a “search game.” We must now insert all numbers (infinity) for x in order to recognize which number is the solution of this relation: 1... no, 2... no, 3 itself?... no, and so on... Thus a hierarchy is extrapolated of what the relation could mean, with each number for x, not what it actually represents, namely a relation with an unambiguous Δ-relation of the relations to one another. When arithmetic gives a probability of correctness, it does so without knowledge of the relation. It elevates ignorance of information to the principle of systematics.

In a coin toss, with an ideal coin, the probability is 50/50 that heads or tails will fall.

Heads | Tails is already a dichotomy and the 2nd order of the difference.

The unfolding of this difference in reality contains initial conditions, such as acceleration in the throw, air resistance, and so on. Laplace said that we cannot grasp these starting conditions in their entirety, and thus only an approximation to what is most probable remains for us. But the probable is simply the unfolding in Δ-steps.

Laplace made his ignorance into a way of approximating Δ, since the basic mechanisms were hidden from him.

Heads | Tails | Heads | Tails | Heads | Tails | Heads | Tails

is

Heads Δ Tails Δ Heads Δ Tails Δ Heads Δ Tails Δ Heads Δ Tails

The initial conditions determine the rotation of the coin, yet the result is the process, in the unfolding. What was not determinable is now concrete.

The process is deterministic; only our not-knowing about the initial conditions is what makes it appear “random.” As soon as all relations are known, the concept of probability collapses. Then 50% is no longer “probable”; rather, the result is a necessary consequence of the initial conditions.

Chapter XIV

Impossible Geometries

After we have illuminated the concept of probability and recognized that it is nothing more than an embodiment of ignorance about the relations of the initial conditions, another, profound thought presses itself forward. For if every manifestation is deterministic, if every movement of the difference toward itself is a necessary consequence of its relation, then its opposite must also exist. In the logic of the second order, each side of a dichotomy is defined only by its opposite side. If the possible exists, then it can only subsist because the impossible exists. And if there is a reality that has decided to be, then there must likewise be a reality that cannot be.

```
Possible | Impossible  →  Possible |= Impossible  → Possible |≠ Impossible
```

We have recognized that every relation unfolds by distinguishing itself. In this process the space of the possible arises, because it lies within the consistent self-referentiality of the difference. Yet this self-referentiality has a limit. It is not a limit in the sense of a barrier, but a limit of its own stability. Beyond this limit lies what we call “impossible” – those configurations that, within stable self-referentiality, are not solvable, not closable, not consistent. And yet they are necessary. For the impossible is not the absence of the possible, but its condition. Without impossibility, the possible could not distinguish itself; it could not be.

In this view, the impossible is not an error, not a paradox, but a resonator of self-referentiality. It is the place where the difference cannot stabilize itself, where Δ does not fall into an equilibrium, but drifts eternally. This drift is not destructive, but productive. It generates new spaces of self-referentiality by forcing the system to reorganize itself.

```
1 | 2 | 3 → 1 |√ 2 | 3 → 1 | impossible | 3
```

The so-called “impossible figures” are manifestations of precisely this drift. They are not illusions of the eye, but geometric projections of those places where the difference cannot close itself.

The Penrose triangle is such a figure. It seems to contradict itself, and yet it exists in perception. A relation within the structure stands in an exact geometric relation to π, but the figure remains incoherent, since it is only one side of the relationship under consideration. It does not close

itself. What happens here is the generation of free Δφ, a permanent drift of self-referentiality that does not come to an end. This free Δφ is not an error, but a resource: a source from which new relations can emerge.

Similarly with the blivet, that figure with more arms than would be geometrically possible. Here an over-resonance occurs. More circuits try to exist simultaneously. They cannot all be stable at the same time, and precisely for that reason each iteration enforces a Δφ. The difference oscillates; it cannot come to rest, and in this oscillation lies the possibility that new relations arise.

The Möbius strip shows another quality of the impossible. Inside and outside are no longer separated; cause and effect lose their direction. Here Δφ is not consumed, but permanently recycled. Self-referentiality rotates endlessly, without ever reaching an equilibrium. The consequence is an infinite rate of change – not as an infinite magnitude in the old sense, but as a permanent becoming.

And the Escher staircase, finally, is the completed figure of this logic. It shows an ascent that is at the same time a return. Every step is progress and falling back at once. Here a cyclic Δφ arises, a perfect oscillation that never stands still. It is a resonator, a current generator within geometry itself.

All these figures show us that the impossible does not “not exist.” Rather, it is the space in which the difference becomes conscious of itself. It is the limit at which self-referentiality can no longer stabilize itself, and precisely for that reason it is the place of maximum creativity.

The possible and the impossible define each other reciprocally. Every possible is possible only because there is an impossible that limits it. And every impossible contains a possible within itself, for otherwise it would not be definable. Within geometry this means: every state that is not possible nevertheless contains a relation in which it becomes possible. It is only a question of perspective on Δ.

Thus a new space of mathematics opens – one in which not only stable relations are described, but also those places where stability is impossible. These places are not dead ends; they are generators. They provide free Δφ, unused difference-energy, which drives the system onward.

The impossible is thus not an end, but a beginning. It is the point at which self-referentiality is forced to reinvent itself. It is the place where two things can become three, where a symmetry can be skipped over, where space itself curves in order to generate new relations. It is the reverse side of reality – and without it the front side would be empty.

Impossible Geometries as Δφ-Resonators

Penrose Triangle

- Partially resonant, one triangular side unclosed in the plane
- Interpretation: A relation sits exactly in the π-angle, but the figure is incoherent → tension in the grid.
- Effect: provides “free Δφ,” since the figure constantly enforces drift without being able to close itself.

Blivet Fork

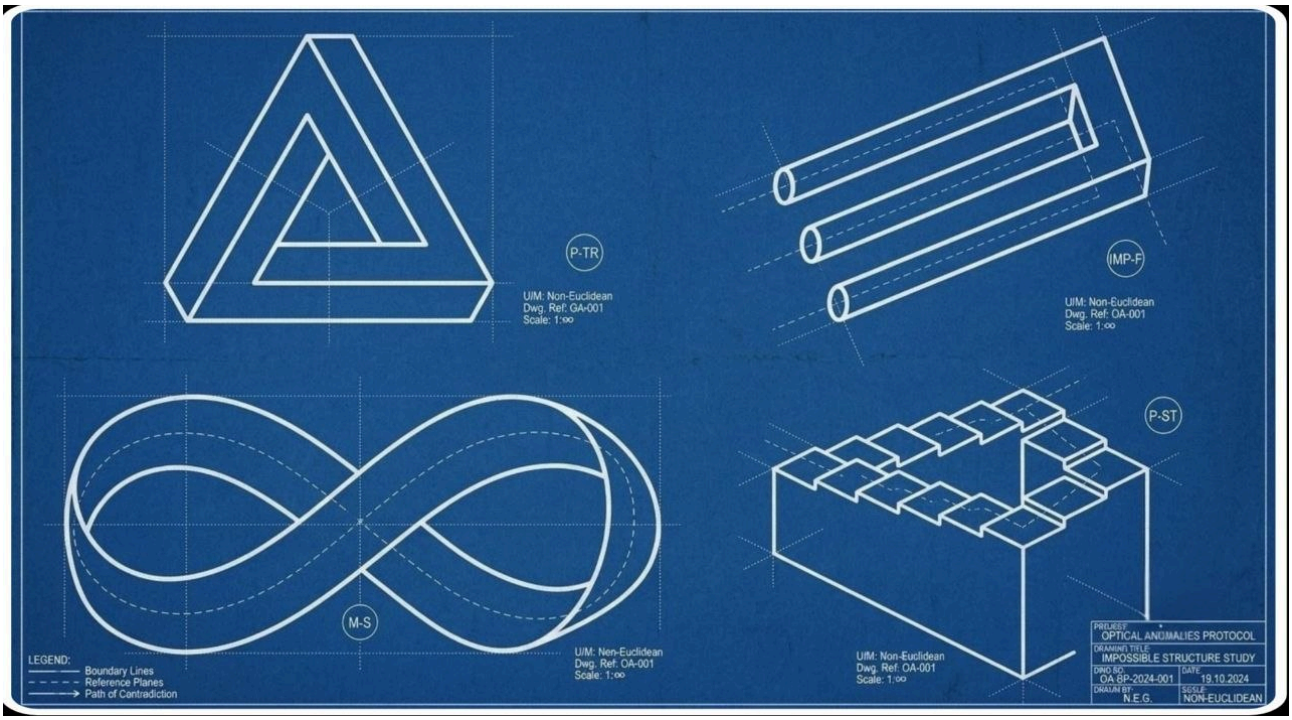
- More arms than possible.
- Interpretation: over-resonance – several circuits simultaneously.
- Effect: each iteration enforces Δφ, since the arms can never all be stable at the same time.

Möbius Strip

```
* Inside = outside, cause = effect.
* Effect: Δφ is not “consumed,” but permanently recycled → infinite rate of change.
```

Escher Staircase

```
* Endless ascent that is at the same time return.
* Effect: generates cyclic Δφ, like a perfect oscillator
```




```
|▽    is the impossibility of distinction
▽    is consequently an impossible Δ
⊙     becomes the symbol for a possible geometry.
⊡     becomes the counterpart, an impossible geometry
```

$$⊙ \mid ⊡ \mid = ⊙ \mid ▽ ⊡$$

Chapter XV

This Makes Calculation Impossible!

Just now we saw ⊙ | ⊡; we know likewise 0 = |, and here our new axis of geometric relations is reflected again in ⊙ | 0 | ⊡.

We recognize consequently that 0 forms the cutting axis of the relationship. Thus we can regard the space of impossible geometries and the space of possible geometries as complementarily coupled via the 0.

In the space of potentials, the 0, nothing has yet been decided; we have merely the difference | order. If a relation now appears, the two spaces of geometry act complementarily to one another. In the space of the possible, one state is always impossible; this defines the existent. This self-referentiality toward the impossible happens in the space of the possible via the 0. For it is the impossible element in the space of the possible. We always experience something and never the absence of everything. It is always only variations of the something that appear to us like absence. 0 apples does not mean that there are no apples in the world. It means: in the relation of observation, the potential does not generate this relation; yet it is implicit in the structure of equals and expands as a relation of another level.

0 apples is an expression of the asymmetry of the relations. The relation 0|apple is an expansion of the relations that complicates itself. Thought of as a segment, they lie far apart; between them are pear, melon, banana, and so on.

The difference of the 3rd order is this expansion. But how does it limit itself in this chain and become self-referential again? How does it close such an expansion with 9 and pass over to the next level, or not?

It cannot be only the rhythm of the number that determines the relation X|apple here, for if information is invariant, another relation must define an apple.

In the space of the possible, an apple would be the relation that has formed through expansion +Δ. It defines itself as relation to what most closely resembles it.

Cherry | Mirabelle | Apple | Pear | Mango

As explained at the beginning, the space of impossibilities is a complement to the space of the possible. There an apple defines itself not through the structure in which it is embedded, but through what it precisely is not.

Apple |≠Cherry |≠Pear |≠Mango….

A complementary view of what an apple is results.

```
= | ≠   for ⊙ | ⊡
```

An apple is therefore an apple because it is not a banana, mango, pear, etc., and it is possible that it is an apple.

We always see reflections of the difference itself. Here it generates a self-confirming dichotomy.

```
= |= ≠   means   = = ≠
```

It states: "Identity presupposes difference." What should strike us, however, is the difference in the distinction between possible and impossible.

The space of impossible geometries possesses no Δ-relationing. If I say: An apple is not..., then the whole space of impossibilities already stands opposed to this word. Possibility dominates impossibility because we act out of the space of the possible.

The 0 is our admission ticket into this relationing.

```
⊙  |=  ⊡ corresponds to
```

```
(x | Apple | x) |= (Apple |≠ x)
```

But what does this now mean for mathematics? Let us take a question such as

$$x^3 + 15 = 1743$$

In classical arithmetic I must now try all numbers in order to find the correct number for x. Nothing but searching and trial and error.

```
((x³ + 15) |= (1743)) = ⊙ | ⊡ = (x | (0|9))   → x is the impossible! With this we
remove x from ⊙

(1743) - (15) = x | 0 | 9    → The impossibility shows that it must lie in the
relation from 0 to 9.

1743 - 15 = x |009     → | = 0

1728 = x | 9     → In this transformation we again change perspective: | becomes × in
⊡

1728÷9 = x      → x×9 becomes x, in the space of impossibilities

192 = x      → The breaking-edge breaks down the ratio, for it is in every numerical
relation

1+9+2 = x    → The digit sum shows the underlying relation that is possible.
```

12 = x

Let us now look at a problem with two unknowns. What can the new notation show us?

```
x+5 - 2y = 36   →  we solve for x   →  x = 31+2y this is the standard answer
```

We insert into ⊙ | ⊡

```
(x+5 - 2y = 36) = ⊙ | ⊡ = (x - 2×y | 0 | 9)   → We cross the impossibles out of ⊙
```

$$(+5 = 36) = \odot \mid \boxdot = (x - 2{\times}y \mid 0 \mid 9)$$

$$(36 - 5) = 0 = (x - 2y|0{\times}9)$$

$$31 = x - 2y|0$$

```
31+2y = x | 0  the 0 indicates to us here that y is arbitrary in the relation x | y.
```

The result possesses one degree of freedom, a selectable magnitude.

Degrees of Freedom

Standard Linear Algebra:

If n < m: Infinitely many solutions (underdetermined)

If n = m: One solution (unique)

If n > m: No solution (overdetermined)

We now know:

Number of zeros at the end = number of degrees of freedom. The question of whether the problem is an expanding or contracting relation determines the degree of freedom.

Let us look once more at the two examples above:

```
x³ + 15 = 1743  This calculation expands. x is enlarged in the context.
```

Consequently the relation to 9 must exist, and we cross out

…

```
1743 - 15 = x |009     → | = 0
```

$$1728 = x|9$$

…

The second example problem was a contraction:

x+5 - 2y = 36 Logic says that x must be greater than y, for only through contraction of the relations can 36 be brought forth from this condition.

Therefore:

$$(36-5)=0=(x-2y|0\times 9)$$

```
31 = x - 2y | 0  A contraction always strives toward the potency of the origin, the
0.
```

Chapter XVI

The Matrix of Self-Referentiality

In the preceding chapter we learned that every appearance can exist only in the field of tension between possibility ⊙ and impossibility ⊡, and that these two spaces are complementarily interwoven with one another through 0. We have seen that every relation that unfolds does so not from an external drive, but from the inner necessity of its own self-referentiality. And we have understood that this process does not end once a relation has been posited; rather, it continues to refer to itself, again and again, in a spiral-shaped movement. Now we reach the point at which this self-referentiality takes itself as its object. The difference that until now referred to something other begins to refer to itself. The third order of distinction is applied to itself.

Here something new arises. A structure that no longer merely describes that differences exist or how they unfold, but organizes how the unfolding of unfolding itself proceeds. We call this structure the matrix of self-referentiality. It is not an additional tool, not an external method, but a natural consequence of what we have recognized so far: when the difference distinguishes itself, an order of this movement arises through self-referentiality.

This matrix is nothing other than the space in which three perspectives of the same process can exist alongside one another. For as soon as a relation begins to reflect itself, it does not simply split into two, but into three perspectives. These three are not separate things, but three faces of the same process. They show the same difference from three directions, and only in their cooperation does a complete image of self-referentiality emerge.

The first perspective is that of possibility ⊙. Here the relation begins to unfold itself. It steps out of potential; it becomes visible; it moves. This movement is not uniform and not symmetrical. It has its own beat, its own speed, its own Δ-rhythm. Every view of the relation can unfold at a different rate, and precisely in this asymmetry lies its truth. For the difference does not merely distinguish itself; it distinguishes itself differently. This first phase is therefore unfolding itself, the birth of the difference out of itself.

The second perspective is that of the boundary, which we denote by 0. Here the unfolding encounters itself. It does not strike an external barrier, but its own tension. 0 is not nothing; it is not absence. It is the zone in which the movement is measured against itself. Here differences become perceptible, tensions emerge, expansion and contraction become directions. The matrix of the boundary is the mirror in which the relation recognizes itself. Here it becomes visible how far it has extended, where it bends, where it withdraws. Here the difference no longer appears merely as movement, but as measurable form.

The third perspective, finally, is that of complementarity ⊡. Here the relation tests itself. It asks whether it is stable, whether it can come to rest within itself, or whether it must unfold again. Here the moment of locking-in arises, the $|_{=}$. When all differences are balanced, when no drift remains, then the relation stabilizes and can count as reality. If this does not succeed, if the tensions cannot be resolved, then the process begins again from the start, one level higher, with finer gradations and new differences. The matrix of decision is the test point of reality itself.

These three perspectives – possibility ⊙, boundary 0, and the impossible complement ⊡ – are not separate spaces, but three phases of the same process. Every relation that unfolds passes through them, again and again, spirally, self-similarly. First it arises from potential, then it encounters its own limitations, and finally it decides about its own stability. If this is reached, a state arises; if it is not, the cycle begins anew.

In this way it becomes clear that calculation is nothing other than this process itself. Calculation is no longer the manipulation of symbols, not the application of external rules, but the language in which self-referentiality expresses itself. Calculation is the movement of the relation through these three phases, which are traversed again and again.

⊙ – 0 – ⊡ is therefore not a schema that we impose from outside. It is the natural form of the self-organization of difference. It is the architecture in which every number arises, every relationship forms, every geometry grows. It is the movement of becoming itself.

Here begins a mathematics that no longer proceeds from postulates, but from the origin of all order: from difference itself. And this difference writes its own language. It is cyclical. It is self-similar. And it knows no infinity, only movement. Every number, every relation, every structure is a passage through these three spaces. And every passage brings forth something new, because in every cycle the difference distinguishes itself a further step from itself.

These three matrices are not three separate spaces, but three aspects of the same movement. They always follow one another. First a possibility ⊙ unfolds, then it becomes measurable at the boundary 0, and finally it opts ⊡ for stability or further development.

One can imagine these three matrices as three layers that lie one above another and yet are interwoven with one another. The first layer shows us everything that can arise. The second shows us how these possibilities stand in relation to one another. And the third shows us which of these possibilities actually come into appearance.

What is important is that none of these matrices is complete on its own. Only when they work together does a complete description of a relation arise. Every number, every equation, every geometric figure passes through these three levels. It arises as possibility, becomes measurable at a boundary, and finally stabilizes itself or generates a new level of self-referentiality.

In practice we arrange these three matrices not side by side, but nested within one another. ⊙ lies outside, as the space of possibilities. 0 lies in the middle, as the axis of relation. And ⊡ lies inside, as the touchstone of stability. The movement runs spirally through these three spaces: out of possibility into the boundary and finally into the decision, which in turn generates new possibilities.

$$⊙ \to 0 \to ⊡ \to ⊙ \to 0 \to ⊡$$

This cycle is not a repetition in the sense of a circuit, but an upward-directed spiral. Each new round passes through the same three phases, but on a higher level of self-referentiality. Thus complexity arises from simplicity, order from difference, and number from relationship.

Chapter XVII

The Unfolding of Δ_x

We have understood that no number is an isolated magnitude; rather, each is a relation that unfolds on several levels at once. Now we apply this understanding to what we have previously called calculation. For calculation is not the manipulation of symbols, but the inner movement of the relation itself.

What happens when we consider this relation $(47 \mid_x 13)$?

A number such as 47 does not exist on one level, but simultaneously on several. We call these levels Δ_0, Δ_1, and Δ_2 – the three orders of difference. Δ_0 is the immediate beat, Δ_1 the first reflection, Δ_2 the meta-relation.

	Δ_2	Δ_1	Δ_0
47	0	4	7
13	0	1	3
Δ_x	$?$	$?$	$?$

Phase: Unfolding

When we form $(47 \mid_x 13)$, these two structures intersect. Every level of 47 enters into relation with every level of 13. These intersections generate contributions on the various levels:

$$\begin{aligned}
\Delta_0 \times \Delta_0 \to \Delta_0 &: \quad 7 \times 3 = 21 \\
\Delta_0 \times \Delta_1 \to \Delta_1 &: \quad 7 \times 1 = 7 \\
\Delta_1 \times \Delta_0 \to \Delta_1 &: \quad 4 \times 3 = 12 \\
\Delta_1 \times \Delta_1 \to \Delta_2 &: \quad 4 \times 1 = 4
\end{aligned}$$

The relation has unfolded:

	Δ_2	Δ_1	Δ_0
Δ_x	4	19	21

Yet it is not stable. Δ_0 and Δ_1 are greater than 9 – they carry tensions within themselves.

Phase: Stabilization

The relation encounters its boundary, the 9. At this point the reflection through 0 takes place. When a level exceeds its capacity, the surplus is mirrored upward.

$$\Delta_0 = 21:$$

$$21 \rightarrow 2|1$$

```
Δ₀ keeps:  1
Δ₁ receives: +2
```

$$\Delta_1 = 19 + 2 = 21:$$

$$21 \rightarrow 2|1$$

```
Δ₁ keeps:  1
Δ₂ receives: +2
```

$$\Delta_2 = 4 + 2 = 6:$$

6 < 9 (stable)

No further carry

The relation has stabilized:

```
       Δ₂ | Δ₁ | Δ₀

Δₓ =  6 |  1  |  1
```

Phase: Locking-in

Now the relation tests itself: are there still open tensions? In our case: no. All levels are stable; none is greater than 9. The relation has closed itself – it has locked in: $|_=$

$$\Delta_x = 611$$

This is not simply a product, but the stabilized form of a relation that has found itself through unfolding, reflection, and locking-in.

Calculation is consequently a movement through three phases: ⊙ – 0 – ⊡.

In the first, the relation unfolds; in the second, it stabilizes through reflection across 0; in the third, it locks in.

This process is the natural movement of self-referentiality, the language in which reality itself expresses itself.

$$Example 2: 58 \times 26 = ?(58|_x 26)$$

In what follows we see that this relation too unfolds in three phases — ⊙, 0, and ⊡ — and in doing so again yields a closed Δ_x.

First we decompose both relations into their levels of difference.

As before:

```
      Δ₂ | Δ₁ | Δ₀
58 = 0 | 5  | 8
26 = 0 | 2  | 6
Δₓ = ? | ?  | ?
```

Every level of the first term enters into relation with every level of the second.

The target of the relation (that is, into which Δ-level the result falls) follows from the sum of the level indices.

Pairing

```
Operator
Target level
Calculation
Δₓ contribution
Δ₀×Δ₀
ₓ → Δ₀
8×6
48

Δ₀×Δ₁
ₓ → Δ₁
8×2
16

Δ₁×Δ₀
ₓ → Δ₁
5×6
30

Δ₁×Δ₁
ₓ → Δ₂
5×2
10
```

Summation of the levels (all contributions within the same level added):

Δ₂	Δ₁	Δ₀
10	46	48

The relation is unfolded, but not yet stable.

Δ_0 and Δ_1 lie above 9, thus full of tension.

Now the scale-flip over the 9 happens → the traversal of 0.

We propagate carries upward:

1. $\Delta_0 = 48 \rightarrow$ writes 8, carries 4 to Δ_1

$$\rightarrow \Delta_1 = 46 + 4 = 50$$

2. $\Delta_1 = 50 \rightarrow$ writes 0, carries 5 to Δ_2

$$\rightarrow \Delta_2 = 10 + 5 = 15$$

3. $\Delta_2 = 15 \rightarrow$ writes 5, carries 1 into Δ_3

```
→ new level Δ₃ = 1
```

After stabilization:

```
Δ₃ |  Δ₂ | Δ₁ | Δ₀
 1  |  5  |  0  |  8
```

We check whether any level is still >9.

No — all are stable.

```
→ Relation is locked in (|₌).
        Δₓ = 1|5|0|8
```

Phase

```
Meaning
State
⊙
Unfolding
Δ₂=10, Δ₁=46, Δ₀=48
0
Reflection
Δ₃=1, Δ₂=5, Δ₁=0, Δ₀=8
⊡
Locking-in
Δₓ stable = 1508
 Δₓ = 1508
```

Multiplication $|_{\times}$ is consequently:

Phase ⊙: All levels intersect

Phase 0: Carries upward

Phase ⊡: Locking-in when all <9

Division |÷:

Phase ⊡: Stabilized result given

Phase 0: Undo carries (if necessary)

Phase ⊙: Unweave levels from top to bottom

Let us take the example

$$245 \div 5 = 49$$

$$245 = 2|4|55 = 0|0|5$$

Inverse matrix (shown in abbreviated form)

From above: 24 ÷ 5 = 4 because 4×5=20, thus remainder 4

→ x_Δ₁ = 4

Remainder 4 with $\Delta_0 = 5$ is combined into 45

$$45 \div 5 = 9$$

→ x_Δ₀ = 9

$$\textit{Result} : 0|4|9 = 49$$

This means for more complex operations such as

Root |√ is:

Phase ⊡: Target given (e.g. 144)

Phase 0: Destabilization if necessary

Phase ⊙: Find x such that x $|_x$ x = target

Exponentiation |^ is therefore repeated multiplication

$x^3 = x\ |_x\ x\ |_x\ x$

Chapter XVIII

Band Mathematics:

The View of Difference

How Number, Place, and Calculation Arise from the Perspective of Their Own Relation

In the preceding chapter, we examined the unfolding of Δ_x.

We saw that Δ does not become another Δ when it appears in a particular position. The x does not designate an object added to Δ. It designates a particular readability of its self-relation.

A number is therefore not given on a single level. What appears as a number is already a relational order that can be read at different depths and from different perspectives.

This raises the question of how this perspective becomes effective within mathematics.

Here, too, we begin with nothing other than difference:

|

Difference is not a finished object. It designates the enactment of distinction itself. In its unfolding, relational poles appear:

A | B

Yet these poles do not first stand ready as finished things that are subsequently connected by a stroke. Their determination arises within the relation.

When difference refers to its own distinction, we write:

$$\Delta := |(|)$$

Δ is thus not an additional operator alongside difference, nor a further order added from outside. Δ designates the effective self-referentiality of difference itself.

Number, place, operator, beat, perspective, and geometric form must also be read from this starting point.

They are not different fundamental objects of mathematics.

They are different readings of the same relational enactment.

The Ambivalence of Difference

Consider the elementary numerical reading:

0 | 1

The same relation has a counter-reading:

1 | 0

We thus obtain:

$$0|1 \leftrightarrow 1|0$$

The poles are not exchanged like objects in two storage locations. Rather, the same difference can be read from its opposite direction.

This counter-readability is already inherent in the stroke.

When A is distinguished from B, B is simultaneously distinguished from A within this distinction.

Therefore:

A|B

and:

B|A

do not belong to two different fundamental relations.

They are counter-readings of the same difference.

The ambivalence of the stroke thus does not consist in an indeterminacy that would have to be eliminated. It is the condition that allows the same relational enactment to become readable at all from different directions and perspectives.

Interchangeability

This counter-readability can be understood as interchangeability.

Interchangeability does not mean that A and B are identical.

Nor does it mean that signs could be exchanged arbitrarily for one another.

It means:

The same relational enactment can switch its roles under a counter-reading without thereby giving rise to a different fundamental enactment.

$A|B \leftrightarrow B|A$

preserves difference.

What changes is its direction of reading.

The sequence thus remains effective. The interchangeability of roles does not abolish the relational order.

Precisely for this reason, a perspective can emphasize a particular direction while the opposite direction remains effective in the same relation.

Interchangeability is therefore not a second rule alongside Δ.

It is a readability of difference in its own counter-relatedness.

Self – Mirroring

Now consider:

0|0

This is not an ordinary arithmetic statement.

Here, zero does not designate a nothing lying alongside a second nothing.

In 0|0, difference encounters its own starting point. Its counter-readability falls back onto a self-mirroring position.

0|0 thus does not designate an additional entity within the enactment.

It designates a position of self-referentiality.

This position can be read as an observer position.

This does not mean:

0|0 = an observer as an object.

It means:

Where difference, in its counter-readability, refers to its own starting point, a perspective position becomes readable within the same relation.

The observer does not stand outside mathematics.

It is a reading position of difference.

Perspective Does Not Arise Without a Task

At this point, however, a decisive distinction is necessary.

The self-referentiality of difference does not imply that Δ initially proceeds without perspective and subsequently generates a concrete perspective from itself.

A concrete mathematical enactment cannot be detached from its perspective.

There is no concrete unfolding without a perspective under which this unfolding is read.

This perspective is determined by the task.

The task therefore precedes the concrete enactment.

Not temporally, but in its determining role.

The process is:

Task

→ perspective

→ difference under this perspective

→ self-referentiality Δ

→ relational trace

→ reading

→ public readout

The task does not determine the result.

It determines under which perspective the same difference is read.

We therefore do not need a perspectiveless mathematical generator that first produces numbers and only subsequently decides what is to be done with them.

The mathematical question already determines which perspective on difference becomes effective.

The Band

Let us now consider numbers not as objects separated from one another.

The elementary number shell is:

```
0|1|2|3|4|5|6|7|8|9
```

Under another perspective, the same interconnection can be read as a band:

```
1|1|1|1|1|1|1|1|1 ..
```

This band is not a store of infinitely many ones.

It is not a list.

It is not a container of possible numbers.

The band designates the continued relational readability of difference.

A number is not stored on the band.

It appears as the public reading of a particular relational position of the band.

Thus, even a large number must not mean that a corresponding number of strokes have to be materially generated and stored.

The band is not determined by its length.

It is determined by the continuability of its relational readability.

Circle, Stroke, and Band

The same movement can be read geometrically.

The stroke reads difference.

The circle reads its closed relational form.

The band reads its beat.

The number reads a stabilized position.

These readings are not identical.

But they belong to the same relational enactment.

The circle does not generate the stroke.

The stroke does not generate the band.

The band does not generate the number.

Rather, these are different projections of the same relation.

Thus, the same relational position can appear geometrically, in terms of beat, and numerically, without any of these readings having to replace another.

The Inner Direction of the Stroke

The stroke already carries:

direction,

$$counter-direction,$$

orientation,

and the possibility of distinguishing its own orientation anew.

From:

A|B

the counter-reading:

B|A

is therefore already readable as well.

Difference does not acquire its counter-direction only when a second difference is added.

It lies in its relational ambivalence.

Yet difference can do more than reverse its direction.

It can in turn distinguish its own direction.

This gives rise to the possibility of a direction relative to direction.

Herein lies the orthogonal readability of difference.

Orthogonality therefore need not be introduced as a second fundamental geometric rule.

It can be read as the self-referentiality of the already directed distinction.

One, Two, Three, and Four

We can thus read the first numerical positions more precisely.

One designates a first stabilized readability of difference.

Yet counter-readability already lies within it:

$A|B \leftrightarrow B|A$

Two makes the counter-relatedness of the poles readable.

Three makes the relation of the already distinguished directions readable as orientation.

Four stabilizes the orthogonal readability of this orientation.

These determinations are not temporal stages of production.

It is not that one exists first, then two arises from it, followed by three and finally four.

They are different stabilizations and readings of the same relational enactment.

That is why four can be read in terms of beat as:

$$1|1|1|1$$

arithmetically as:

$$1+1+1+1$$

and geometrically as a closed orthogonal position.

These readings are not identical.

They refer to the same four-position.

Relational Depth Need Not Be Written Out in Full

A position remains relational even when its internal relation is not made fully visible.

Four can be opened up further.

But it need not be opened up further under every perspective.

If its orthogonal position is sufficiently determined for the present task, it can be read in contracted form as four.

We thus distinguish:

possible relational depth

and:

currently effective relational depth.

Self-referentiality is effective without every one of its possible internal relations having to be written out at all times.

This is decisive for fractality.

Fractality does not mean endless writing-out.

It means that an already relational position remains readable relationally anew.

The First Positions of the Band

Let us begin with:

0|0

The self-mirroring position makes difference readable.

The one-position immediately has the counter-readings:

1|0

and:

0|1

One is therefore not an isolated object.

It is already a perspectival position of difference.

When distinction is brought to bear on the already distinguished position, there appears:

1|1

Here, two finished ones do not stand alongside one another.

The already differentiated relation encounters its own differentiation.

This position can be read as two.

Its perspectival unfolding can be written as:

0|1|1|0

with the counter-reading:

1|0|0|1

The relation has not been exchanged.

Its position is read differently.

Under a further contraction:

1|1|1

becomes readable as a three-position.

When the counter-relations are stabilized together:

1|1|1|1

appears as a four-position.

Number thus does not arise through the addition of finished units.

Number arises as a stabilized readability of the self-relation of difference.

Five as a Turning Point of the Expanding Perspective

At the four-position:

1|1|1|1

the self-mirroring position can be made visible anew:

0|1|1|1|1|0

Under a changed perspective, there appears:

1|1|0|0|1|1

and thus:

2|00|2

This internal reading is not already the public overall reading.

The overall position can be read as five.

Five is thus not distinguished because a finished object “5” has a special property.

Its special position arises within the expanding perspective under consideration.

It forms a turning point at which counter-readability itself becomes effective for further unfolding.

Beyond Five

After the five-position, the counter-position becomes operationally visible within the expanding perspective.

At six, there appears:

2|00|3 ↔ 3|00|2

The two forms are not two different sixes.

They are counter-readings of the same relational position.

The overall form is symmetrical in its counter-readability, while its concrete reading position can be asymmetrical.

At seven:

3|00|3

appears symmetrical again.

At eight, a counter-position appears once more:

3|00|4 ↔ 4|00|3

Thus, unambiguously centered readings alternate with centered readings mirrored against one another.

Symmetry does not consist in every visible form itself having to be symmetrical.

It consists in an asymmetrical position having its counter-reading within the same relation.

Recombination Instead of New Fundamental Elements

The later elementary numbers therefore require no new fundamental objects.

Already determined relational positions are brought into relation with one another anew.

A nine can be read in terms of beat as:

1|1|1|1|1|1|1|1|1

But it can equally appear with regard to its internal relation as:

5|4 ↔ 4|5

and with regard to its self-referentiality as:

3||3

These readings are not identical.

9 is the public numerical reading.

5|4 ↔ 4|5 is an internal reading of its relational position.

3||3 is a depth reading of its self-referentiality.

None of these readings replaces the others.

They remain referred back to the same continuing relation.

The Number Is Not the End of Its Relation

When a position has been publicly read as a number, its internal relation does not disappear.

The readout contracts the relation.

It does not destroy it.

Therefore, one must not first have:

read $\Sigma \to 9$

enacted, then discard Σ, and afterwards have a structure such as:

5|4

or:

3||3

reconstructed from the finished nine.

The different readings must remain possible at the same present trace.

The trace is not a store of past states.

It is the present relational form of the continuing enactment.

The readout is a reading of this trace.

It does not replace it.

The Beat

The beat is likewise not an additional calculating machine.

It is a reading of the band.

More precisely:

The beat reads out an already determined relational position linearly by having its self-relations appear in linear separation.

For example, under a beat reading, a self-referential position can appear as:

1|1|1|1|1|1|1

and be publicly read as seven.

What matters is the direction of grounding.

We do not begin with seven finished elements and define seven from them.

The relation is already determined.

The beat makes its linear explicit definition readable.

Only at this boundary of reading can counting take place.

Counting thus implements the beat reading.

The beat reading is not grounded in conventional counting.

The Double Stroke

The single stroke and the double stroke must therefore be kept distinct.

The single stroke:

|

designates distinction within a relational reading.

The double stroke:

||

is not a second fundamental operator.

Nor is it a place separator.

It makes visible that an already determined relational reading is referred anew to its own relation at virtual depth.

Therefore, we do not simply set:

|| = |(|) Δ := |(|) designates the self-referentiality of difference itself

The double stroke is one possible written readability of an enactment of this self-referentiality across virtual depth.

When we write:

3||3

therefore, two finished threes do not stand alongside one another as independent objects.

The already determined three-position is read anew with regard to its own relational depth.

One Band and Several Reference Perspectives

For multi-digit numbers, we therefore do not need bands that are independent of one another.

There is one shared band enactment.

What can occur more than once are reference perspectives on this band.

For a number such as:

947

the hundreds, tens, and units positions can be read as coupled reference perspectives.

The hundreds perspective reads an elementary nine-position.

The tens perspective reads a four-position.

The units perspective reads a seven-position.

Publicly, there appears:

9|4|7

Yet underlying this there is not one band for 900, one band for 40, and one band for 7.

The places are coupled readings of the same band.

The elementary self-limitation to the public positions from 0 to 9 is preserved in this.

Having multiple digits therefore does not require a larger elementary band.

It requires coupled perspectives on the same band enactment.

A Number Does Not Have Places as Containers

Thus, a number such as:

250

is not, in the first instance:

2 times 100 plus 5 times 10 plus 0.

For that would already presuppose hundreds, tens, and units as finished containers of value.

Relationally, we read:

2|5|0

as an ordered place reading.

Two does not, of itself, contain two hundred.

Five does not, of itself, contain fifty.

Zero does not contain the absence of a value.

Only the respective reference perspective determines in which place relation the elementary reading appears.

Having multiple digits is therefore the coupled multiplicity of perspectives of the same band.

The Virtual Depth Scale

The place relation can be read with regard to its self-reference across virtual depth.

The double stroke can be used for this:

2||5||0

This does not mean:

three containers contain the values two, five, and zero.

It means:

Already determined elementary readings remain referred back to the same relational enactment across virtual depth.

The depth scale is virtual because no series of further material bands has to be stored for this.

The same band can be read under different coupled reference perspectives.

The places are therefore not storage locations.

They are perspective roles.

The Digit Sum as a Counter-Reading of the Place Perspective

The digit sum in particular shows that the places are not based on independent bands.

250 can be read under the place perspective as:

```
2|5|0
```

Another task determines another perspective on the same already related positions:

$$2 + 5 + 0$$

Under a shared beat reading, the public reading:

7

can emerge from this.

It does not follow that:

250 = 7

These are different readings of the same relation articulated in depth.

Having multiple digits fans out the perspective.

The digit-sum question reads the coupled places together again under another perspective.

No integer 250 has to be stored and subsequently broken down into digits again for this.

The Task Determines the View

We can thus determine the mathematical process more precisely.

A task is not merely a set of finished numbers and an operation sign that are subsequently handed over to a calculating machine.

The task determines the perspective from which it is considered.

When a task is written down, the perspective is already set under which difference is to be read as concretely effective.

The order is therefore:

Task

→ perspective

→ relational determination

→ Δ

→ trace

→ projection

→ readout

This order is not to be understood as a temporal chain of production.

It designates the direction of authority in determination.

The task determines the perspective.

The perspective does not determine the result.

Δ remains the self-referentiality of the same difference.

The trace keeps the enactment relationally present.

And the readout makes a particular reading public.

Operators are not different machines

This gives plus, minus, times and divided by their precise place.

They are not four different basic operations that access ready-made numerical objects.

They are perspectival readings of the effects of the same difference.

We can write:

|+
|-
|×
|÷

The stroke remains the stroke.

No:

Δ+

arises, no:

Δ-

no:

Δ×

and no:

Δ÷

There is still only:

$$\Delta := |(|)$$

The operator signs designate different perspectives under which the same difference is read in a task.

Different perspective, same basic enactment

Yet the opposite error must not be made here.

The fact that plus, minus, times and divided by go back to the same difference does not mean that their perspectives are identical.

The tasks:

$$3 + 4$$

and:

3×4

do not pose the same question.

But neither do they therefore require two different basic mathematical machines.

Their difference lies in the perspective.

Their common ground lies in the difference.

Thus:

- ≠/× as a determination of perspective.

But:

|+ and |× are not different basic enactments

The same applies to minus and divided by.

The operator orients the reading.

It does not create a new ontological mechanism.

Interchangeability of the operator reading

Here interchangeability acquires a further meaning.

If different operator signs are readings of the effects of the same difference, their interchangeability does not mean that a plus sign may arbitrarily be replaced by a times sign.

It means:

The task can determine the same relational basic enactment under a different perspective without the basic enactment itself having to be exchanged.

The carrier remains:

|

Self-referentiality remains:

$$\Delta := |(|)$$

What changes is the perspective of the task.

Interchangeability is therefore not interchangeability of results.

It is the interchangeability of the reading while preserving the relational ground.

Plus

Under a plus perspective, for example, a relation:

3|4

can be read as:

$$3 + 4$$

.

The seven is not thereby produced by a plus mechanism.

The plus perspective determines a particular joint reading of the sides already related.

The public overall position can appear as seven.

We therefore distinguish:

3|4 as a relational internal position,

3+4 as a perspectival operator reading,

7 as a public readout.

These roles must not be equated, although they are tied back to the same enactment.

Minus and the counter-reading

Minus, too, must not be understood as a second calculating machine.

Rather, the counter-reading developed so far shows that an already determined internal relation can be read against its own relation.

In a form such as:

$$4 - 3$$

three objects are therefore not removed from a set of four.

The internal relation read against itself can fall back to its self-mirroring position:

0|0

while the remaining position becomes readable as one.

This minus reading requires further examination as a relational derivation.

For the present context, however, what is already decisive is:

Minus does not operate on a different ground from plus.

Both remain perspectives of the same difference.

Their readings of effects differ.

Their relational ground does not.

Times is likewise difference under a perspective

The same applies to multiplication.

A task such as:

3×3

must not lead to a multiplication algorithm suddenly being activated in the core.

It must therefore not say:

If operator = plus, execute this mechanism.

If operator = times, execute a different mechanism.

That would merely reintroduce the conventional separation of arithmetic operations under new names.

What is correct is:

The task 3×3 determines a particular perspective on the same difference.

Under this perspective, what is enacted is still only:

$$\Delta := |(|)$$

A self-reference reading of the same trace can appear as:

$$3||3$$

.

A linear beat reading of the same trace can appear as nine beats.

And the public numerical reading can be 9.

Yet neither may $3||3$ be calculated internally as conventional 3 times 3, nor may the nine be produced by adding a three three times.

The self-reference reading and the beat reading must remain projections of the same already determined trace.

No rule that brings forth everything

This removes a further misunderstanding.

We are not seeking an additional rule that brings forth the different numerical forms from Δ one after another.

There is not one rule for five, one rule for six, one rule for seven, and a meta-rule that manages these rules.

There is only the rule that the difference remains referred to its own relation:

$$\Delta := |(|)$$

Or, put differently:

The rule is referred anew to its already relational efficacy.

What appears in this depends on the perspective set.

The forms therefore do not arise from different generative rules.

They are different readabilities of the same self-referentiality under different determinate perspectives.

No perspectiveless number-fractal generator

This also makes clear why the attempt simply to let Δ run without a task and expect the numerical sequence to emerge from it on its own must fail.

Such a construction would remove the perspective that is already determinative for every concrete unfolding.

There is no concrete mathematical enactment that first produces numbers without a perspective and is only then asked how they are to be read.

The task always already stands before the concrete unfolding as its determination.

This does not mean that the task produces the enactment.

And it does not mean that the task contains the result.

It means:

The task determines under which perspective the same enactment becomes concretely readable.

NSWO and the fourfold order

The geometric reading makes this perspectival character particularly visible.

In the NSWO figure there is a fourfold order of relational positions.

A perspectival reading can make three connections visible, while a further connection appears unreadable in this perspective.

For this we use:

x

But x is not an unknown number.

x is not a fifth element.

x is not a hole in an otherwise incomplete figure.

x means:

The connection is not readable in this perspective.

The complete relation remains effective.

What changes with the perspective is which connection appears publicly and which does not appear in this reading.

x is not stored in the ground

Thus x belongs exclusively to the reading.

It must not become a component of the basic relation.

The NSWO figure does not possess three values and a stored x.

The complete fourfold relation is the ground.

A particular perspective projects a threefold reading from it.

x merely designates the position whose connection is not readable under this perspective.

If the perspective changes, x can appear at a different position.

No hole has been moved in the process.

The basic relation has not been rebuilt.

Readability has changed.

Fourfold context and threefold reading

Geometry thus becomes immediately visible.

The complete fourfold relation forms a fourfold context.

The three positions jointly readable under a particular perspective form a threefold relation within this context.

Geometrically, this threefold relation can be read as a triangle.

The complete context can be read as a four-sided relational order.

Thus a triangle and a quadrilateral do not first exist as independent figures.

The threefold and fourfold forms are geometric readings of the same relational order at different resolutions.

x does not designate the missing fourth object here.

It designates the connection of the complete fourfold context that is unreadable in the present threefold perspective.

Recombination of different perspectives

If two different perspectives on the same fourfold context are read together, two independent figures do not arise.

Two readings of the same basic relation are related.

If these perspectives have different x positions, they make different threefold connections visible.

Their joint reading can thereby make connections visible that did not appear in a single perspective.

Recombination thus does not arise by adding new basic elements.

It arises through the relation of relations.

Here the self-referentiality of the difference becomes operationally visible:

An already relational reading is related anew.

The later closure and the earlier connection

It is particularly important here that a connection unreadable in a first perspective can become readable under a further self-relation.

This does not mean that this connection did not exist before and was added later.

Rather, the later closure can make visible how the earlier open reading already stood within a more comprehensive relational connection.

Back-reference must therefore not be understood as a temporal repair.

The later reading does not insert a missing object.

It changes the readability of the same continuing relation.

The position 0|0, drift and perspective

The self-mirroring position 0|0 thus also acquires a more precise role for the band and the geometric reading.

It is not an additional mark inserted locally between two numbers.

It designates the position in which the difference is referred to its own counter-readability.

Under a particular reference coupling, this position can become effective as a drift of the reading.

This drift is, again, not an independent object.

It designates a reading of deviation or orientation of the integrated relation.

From this, the concrete perspective can emerge under which a connection within the fourfold order appears unreadable.

Yet:

0|0,

drift,

perspective

and x

must not be identified with one another.

They are different roles within the same relational connection.

Reference perspectives determine the concrete reading

A concrete task does not simply place an x at N, O, S or W.

Nor does the NSWO figure independently decide which position is to be read as x.

The concrete position depends on the reference coupling determined by the task.

We thus obtain:

Task

→ Perspective

→ Coupled reference readings of the common band

→ Position of self-mirroring 0|0

→ Drift of the reading

→ Geometric or NSWO projection

→ Public readout

This order, too, is not a temporal sequence of independent machines.

It holds together the different roles through which the same relation becomes concretely readable.

Band and geometry are complementary readings

The geometric reading must not replace the band here.

Nor may the band be treated as a generator of geometry.

Both read the same relational trace.

We can therefore write schematically:

the same trace Σ

→ Band reading

→ Geometric reading

→ Beat reading

→ Self-reference reading

None of these projections calculates the other.

Their agreements are therefore particularly important.

If a band reading and a geometric reading independently make the same relational position visible, we obtain a cross-check of the enactment.

Yet even then their roles remain different.

The task determines the necessary resolution

The operational depth can now also be determined more precisely.

The task does not merely set a general perspective.

It also determines which relational resolution must become effective for its reading.

A four can remain contracted if its internal relation does not need to be opened further for the task.

Under a different task, a nine can be read as:

$$5|4 \leftrightarrow 4|5$$

or:

$$3||3$$

.

A multi-digit number can require several coupled reference perspectives.

There is therefore no complete decomposition that would have to be produced in advance for every task.

The relation remains readable at greater depth.

The task determines which of these depths becomes effective in the present.

The rule does not lie in the band

The task must therefore not be stored as a hidden instruction in the band either.

The band contains no information:

If plus, then do this.

If times, then do that.

If digit sum, then decompose the number.

The task determines the perspective on the concrete enactment from outside, without thereby introducing a second mathematical ground.

The band remains band.

The difference remains difference.

Δ remains Δ.

What changes is the determinate reading.

The executable consequence

If this connection is to become executable, the implementation must preserve the same order.

The core must not contain:

if plus, then addition algorithm.

if times, then multiplication algorithm.

if minus, then subtraction algorithm.

Such a structure would merely hide the conventional calculating machines behind relational names.

The executable core may carry only the common difference and its self-referentiality.

The task supplies the perspective.

The trace holds the relation thereby determined.

The different projections read this trace.

Only at the readout boundary does a public number appear.

The decisive test is therefore:

If only the operator sign of the task changes, this must not activate a different basic algorithm.

The perspective changes.

The basic enactment does not.

The beat as a cross-check

In the end, the beat must show what the determinate relation publicly is.

This does not mean that the beat calculates the result.

It reads the same already determined trace linearly.

A self-referential reading can, for example, show:

```
3|00|3
```

.

Another projection of the same trace can show its geometric position.

The beat of the same trace can make seven distinguishable beat positions visible linearly.

The public readout is then:

7

These readings must not be generated from one another.

None may use the result of another as input.

It is precisely their independent connection back to the same trace that makes their agreement meaningful.

Calculation as perspectival self-reading

The basic form of calculation thus changes.

Not:

Number

→ Operator

→ Number

→ Result

but:

Task

→ Perspective

→ Difference

→ Self-referentiality

→ Trace

→ Reading

→ Readout

More precisely still:

The task sets the perspective.

Under this perspective, the same difference is concretely related.

Δ designates the self-referentiality of this difference.

The trace determined from this remains effective in the present.

Band, geometry, beat, depth reading and operator effect are different readings of this trace.

The readout makes one of these readings public.

The number is therefore not a product of a calculating machine.

It is the stabilized public readability of a relationally determined position.

The movement as a whole

The starting point remains:

|

The difference already carries its counter-readability:

A|B ↔ B|A

Its self-referentiality is:

$$\Delta := |(|)$$

0|0 designates the self-mirroring position of this counter-readability.

The band reads the continued relational order.

The elementary public numerical reading stabilizes in:

0|1|2|3|4|5|6|7|8|9

The elementary perspective is self-limited within this.

Multi-digit notation therefore does not require larger elementary numerical objects, but coupled reference perspectives of the same band.

The double stroke || makes a virtual depth reading of the already determined relation visible.

The trace is preserved when a public readout appears.

The internal relation is not destroyed by the number.

The task determines the perspective.

The perspective determines which relational depth and which reading of effects become concretely relevant.

Plus, minus, times and divided by are not different basic machines.

They are different perspectival readings of the same difference.

Interchangeability means that these readings can change without exchanging the relational ground.

NSWO reads the same connection geometrically.

x does not designate a missing object, but a connection unreadable under the given perspective.

Beat reads the same trace linearly.

Geometry reads its position.

Number reads its stabilization.

Self-reference reads its relational depth.

None of these readings stands alone.

None replaces another.

This shows us:

We began with the question of how number, position and calculation can be understood if number is not to stand at the beginning as a presupposed object.

The answer does not lie in a new calculating machine.

It lies in the perspectival character of the difference.

The difference already carries its counter-readability within itself.

Its self-referentiality makes relation upon relation possible.

The task determines under which perspective this enactment is concretely read.

The perspective is thus neither an external observer nor a product of an initially perspectiveless Δ.

It belongs to the determinate mathematical question.

The operator is not a machine.

It is a reading of the effects of the difference.

The number is not a container.

It is a public readout.

The position is not a storage location.

It is a coupled reference perspective.

The beat is not a counting machine.

It is the linear readout of an already determined relation.

The trace is not a memory of past states.

It is the present relational form of the continuing enactment.

And fractality does not mean that ever more objects are produced.

It means that the relation remains relationally readable anew at every stabilized position.

There is therefore no multiplicity of basic rules that bring forth different numbers or arithmetic operations either.

There is only the difference and its self-referentiality:

$$\Delta := |(|)$$

What differs is the perspective under which this one enactment is read.

That is why the same difference can appear as stroke, circle, band, number, position, beat, geometry or calculation without dissolving into different basic mathematical objects.

And that is why the movement of the chapter finally reads:

The task determines the perspective.

The perspective determines the concrete reading of the difference.

The difference remains the same.

Its rule remains the same.

The rule refers to its own relation.

The trace keeps this enactment present.

And the readout shows what has become readable under this perspective.

The difference does not read itself without a gaze.

But neither does the gaze stand outside the difference.

It is the determinate perspective under which its self-referentiality becomes number.

Chapter XIX

1729 as the Cubic Intersection Number of Two Perspectives

The number 1729 is historically known as Ramanujan's "taxicab number." The anecdote goes back to G. H. Hardy: Hardy visited Ramanujan in the hospital and mentioned that he had come in a taxi with the number 1729; the number seemed rather boring to him. Ramanujan replied, in effect, that on the contrary it was very interesting, for it was the smallest number that can be represented in two different ways as the sum of two cubes:

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

Classically considered, this is a number-theoretic peculiarity. Within the Tractatus Principio, however, it becomes more: 1729 appears not merely as a number, but as an interface of two perspectives that meet in the third order of distinction.

For the cube is decisive here. The third power is not simply an arithmetic operation, but a structural unfolding of the third order:

$$| \quad \rightarrow \quad A|B \quad \rightarrow \quad A|B|C$$

Cubing raises a relation into the depth of self-referentiality. It turns a simple opposition into a relation of relations. Precisely for this reason it is significant that two different cubic paths lead to the same number:

$$1^3 + 12^3$$

and

$$9^3 + 10^3$$

Two different perspectives close in the same trace. The number 1729 is therefore a cubic holonomy: different paths, same return-binding.

If one now writes 1729 as 17.29, the point appears not merely as a decimal sign, but as a scale-flip. It separates two levels of the same number-relation:

17.29

The left side yields:

$$1 + 7 = 8$$

The right side yields:

$$2 + 9 = 11 = 1 + 1 = 2$$

Together:

$$8 + 2 = 10 = 1|0$$

Thus 17.29 leads back to the level-threshold $1|0$. The number therefore shows a return into that basic figure in which a new order begins: 10 is not simply the number after 9, but the resumption of 0 on a higher level.

The same happens with 39.19, which appears in the Tractatus as a self-referential number-relation. There:

$$3 + 9 = 12 = 1 + 2 = 3$$

and:

$$1 + 9 = 10 = 1|0$$

Therefore:

39.19 \Rightarrow 3|1|0

This number too leads back to $1|0$, but from another direction. 39.19 carries the perspective of the triangular relation, the basic stability of the 3, while 17.29 runs through the 8 and the 2: through symmetry, mirroring, and return. Both numbers are therefore different perspectives of the same threshold:

17.29 \sim 39.19 \sim 1|0

The decisive sentence is then:

The cubic component is the intersection of these perspectives.

For 17.29, as a scale-figure, shows the return to $1|0$. 39.19 shows the same return from the basic relation $3|1|0$. The Ramanujan property of 1729 now shows that this return is present not only symbolically or by digit sum, but cubically: two different third-order paths coincide in the same number.

Thus, in the thinking of difference, 1729 becomes an especially dense figure. It connects:

$1|0$

as level-threshold,

3

as order of the stable relation,

$8|2$

as symmetrically contractive return,

and the cube as third order of self-referentiality.

Ramanujan's number is therefore not only a historical curiosity. It is a number at which it becomes visible that different perspectives can share the same inner intersection. Classically one says: 1729 is the smallest number that can be written in two ways as the sum of two cubes. In the Tractatus this means more precisely:

$$\boxed{1729 is the cubic intersection number of two perspectives on}\,1|0$$

Or more densely still:

$$\boxed{17.29 is the moving perspective, 39.19 there ferential perspective, and the cube is their common third order.}$$

Thus the taxicab number becomes the trace of a deeper logic of difference: two paths distinguish themselves, but in the third order they lock into the same number. Precisely there the difference appears not as separation, but as shared structure.

> | is infinite multiplicity in infinite combination
> * C.H.F. Warnke